Sharing an Innovative Mathematics Teaching Experience

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When taking the Major Method Core in P.C.Ed. (part-time) course at the University of Hong Kong last school year (1995-96), I came in contact with some insightful methods on classroom teaching. One of the innovations which appealed to me was the use of discussion between pupils to help them have greater understanding of the mathematical concepts or topics in mathematics they learnt in the two school terms. So I decided to find out more about discussion-based teaching by using an approach known as '*action research*'. Action research is a systematic investigation of classroom teaching throughout one whole school year or across school years undertaken by teachers themselves.

To be able to learn more about the use of discussion, I needed to be fully aware and self-critical of my own teaching practice, strategies and skills as well as to be ready to have gradual refinement of them if necessary throughout the whole school year. My role was to be 'an active researcher as a teacher'. Unlike traditional academic research where teachers and researchers are *not* the same persons, I not only became an active participant in classroom teaching, but also turned to be a first-hand researcher in such practice. I had to make decisions very quickly, closely observe students' cognitive processes of learning, realise their learning difficulties and subsequently make prompt responses or devise methods, strategies to apply the theory in a way which responded to the situation concerned. In short, the posts of *researching* a teaching situation and of *acting* on it were conjoined together.

For each teaching topic, some sort of discussion-based learning was carried out whenever possible. Some of the questions I needed to answer during the action research investigation were what kinds of questions were most effective in prompting discussion, how long and how often these sessions should be. For the eleven topics in Secondary 4 mathematics in the whole school year, I *alternated* the discussion-based approach. This means that for each topic, about half the teaching was done through discussion and half through the knowledge-delivery way. I used this approach in my two

Secondary 4 classes. There were three types of student-directed discussion which were utilised as follows:

(i) *pair discussion*
(ii) *group discussion* usually in groups of four to six
(iii) *half-class discussion* which proceeded between the two-halves of the whole class

Procedure for discussion-based approach

Students (in each pair, group or half-class) were given some exploring questions found in worksheets. They needed to accomplish the tasks given in the questions through discussions within a time interval. Afterwards, a representative (chosen from the pair, group or half-class) had to summarise its standpoints or present its solution to the whole class. Finally, I laid down some overall comments on those summaries or solutions.

For instance, for the introductory topic about quadratic curves in a double lesson, students began by answering an open, exploring question concerning the daily application of quadratic curves and their special features. Students were divided into groups and spent 5-10 minutes brainstorming some best-fit examples. They were the physical outlook of some artificial building (like the Cultural Centre in Tsimshatsui), the projectile of a ball of the 'jump-over-the-rope' game during childhood.

Later each group was asked to investigate how the values of a, b, c affect the shape of a quadratic curve in the general form:

$$\mathbf{y} = a\,\mathbf{x}^2 + b\,\mathbf{x} + c \quad (a \neq 0).$$

In each group, some put forth seemingly plausible hypotheses such as:

(H_1) : the positive value of a affects the shape of the curve;

(H_2) : the integral value of a affects its shape.

The other members were required to confirm or falsify the hypotheses by giving good explanations or relevant counter-examples like:

Graph 1 : $\mathbf{y} = 1\,\mathbf{x}^2 + \mathbf{x} + 2$ and Graph 2 : $\mathbf{y} = 1/2\,\mathbf{x}^2 + \mathbf{x} + 2$ confirm (H_1),

whilst Graph 1 and Graph 3 : $\mathbf{y} = -1\,\mathbf{x}^2 + \mathbf{x} + 2$ falsify (H_2) .....

After group discussion, one chosen representative of each group presented some significant results, insights or findings to the whole class.

Finally, wrong ideas or illogical reasoning would be pinpointed and amended whereas correct, creative ideas or valid reasoning would be mentioned and appreciated. Worksheets finished by each group had to be handed in to me after the lesson.

My responsibility as a teacher during discussion

I ought to have a superior grasp of the subject matter in order to recognise and focus on significant things brought out by the students through constructive discussion. In the meantime, I had to realise when students were at the end of a blind alley or any discussion needed to be redirected.

For example, in one group discussion, students were asked to find the nature of irrational numbers as well as how they differ from rational numbers. Hints, like decimal conversion, were given to students at the beginning. On one hand, some group members seriously doubted the existence of the non-repeating decimal nature of irrational numbers. They tended to think that an irrational number might have a group of repeating digits even if the number of decimal places involved was unpredictably large. On the other hand, those who wanted to clear up the doubt had great difficulties in finding any obvious example of an irrational number in non-repeating decimal.

At that time, I took my turn and participated in the subsequent discussion by suggesting a notable example:

> " 0.10100100010000.... (after each fixed '1', the number of zeros is increased by 1 each time)"

Then I turned their attention to the non-terminating, non-repeating character of the decimal for π (the ratio of the circumference to the diameter of a circle), ... and finally the non-repeating character of decimal representations of irrational numbers.

Assessment of the approach

After students had learnt a topic, I asked them to complete questionnaires concerning whether they thought the discussion approach had helped their learning and understanding of that topic. Students' own learning problems, personal feelings about the lessons or individual comments on the approach were recorded in some informal interviews held before or after school hours. Furthermore, students in each class could use a logbook to air their views on the previous lessons. Wherever they found

any shortcoming or limitation of the approach, they were allowed to mention it and thereby make suggestions freely. Whenever they found difficulties in understanding the teacher's viewpoints or in doing any exercises, they could write down such stuff in detail in the logbook.

For example, early in the year when the approach was new, some of the students wrote:

(i) "Only a small proportion of the whole class (usually the smart ones) really took part in previous discussion whilst the others just chattered around the desks."

(ii) "What a noisy, disturbing learning environment!!! It was very difficult to concentrate on the discussion."

(iii) "Guidelines given in the worksheets were not straightforward and often involved a lot of things that we don't understand."

(iv) "Teacher's supervision was not enough. He helped only one group each time."

(v) "We couldn't complete the tasks in worksheets in time."

(vi) "After heavy lunch, we nearly fell asleep. How could we join in any discussion!!!"

(vii) "We were bored with those familiar contents whereas we were unclear about some topics that we haven't learnt thoroughly."

Notably, at that time, I was able to make many valuable modifications from these comments.

Throughout the whole school year, the students' learning processes roughly fell into 3 stages. For the first two months, the students found interest in the discussion-based approach but were not fully adapted to such new classroom setting. At the very start, many under-achievers (occupying 60-70% of the whole class) had no motivation in learning and showed no progress in learning. Yet after the discussion-based learning approach had proceeded for nearly 2 months, their learning attitudes drastically turned to be more positive. Their questionnaires and interviews showed that they even started to appreciate the approach and took an active role in leading or joining discussion afterwards. They said they discovered that when they had learnt a topic through discussion, they were less likely to forget it and were beginning to build up their academic interest (in particular, their facial expressions and feedbacks were warm and enthusiastic).

Between December and March, I found myself in a difficult time while both students and I accepted and made adjustments to the approach. I still

believed in the value of discussion and wanted to persist, but there were times when it would have been easier to give up. Even though students and I tended to accept the approach, I found that there were shortcomings in it. Its application was gradually modified as follows.

(a) I found that the time for student-directed discussion should better be shortened (e.g., 10-15 minutes for each single lesson on class discussion; 20-30 minutes for every double lesson on group discussion.) The students could not sustain a discussion for any longer than this. It was not successful to try to use discussion immediately after lunch, after an examination or after long holidays. Inappropriate length or timing of discussion often led to many serious class discipline problems, an unavoidable increase in teacher's workload and students' sudden loss of attention.

(b) By comparing the students' performances in tests, quizzes and examinations on the topics taught traditionally and those taught through discussion, I realised that new basic definitions, simple deductions and calculations should better be taught traditionally. Only in gaining conceptual understanding, problem-solving skills and formulating mathematical proofs and investigations could discussion-based learning be best employed.

(c) Besides discussion-based learning, other viable strategies needed to be carried out. For example, constant revision of topics could ensure that students kept a good memory of the lesson contents and knew how to solve related problems correctly. Moreover, frequent tests, quizzes and examinations could effectively check the level of their understanding.

(d) Furthermore, discussion questions of different levels of difficulty should be given to students of various abilities so that they could all freely develop their unique learning processes and realise their potential.

The third stage, from April to June, was one of maturity. The students had some particularly successful learning experiences. For some topics where logical reasoning or argumentation is required, students' overall performance in one class was unexpectedly good. They even showed improved results in subsequent tests, quizzes and examinations. For instance, in the lessons concerning the geometry of circles, the students were

able to formulate some geometrical statements or proofs consistently and logically through discussion.

Some fruitful results which did not often come form exclusive use of school textbooks were made by students through discussion. For instance, some students in group discussion found that:

> The variable x in some direct variation cases such as:
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> $\mathbf{y} = k\mathbf{x}$ (k is a variation constant) is restricted to be *a natural number* when applied to some daily life cases like the relationship between the number of oranges bought (**x**) and their overall cost (**y**) without any bargain sale or price.

> Some of them even discovered that:
>
> the mere criterion 'when **x** increases, **y** decreases, whereas **y** increases when **x** decreases' does not sufficiently justify the case where a simple inverse variation relationship holds between **x** and **y** governed by the equation : $\mathbf{xy} = k$ (k is a variation constant).

I discovered that it was important to base discussion questions on students' knowledge, even if this was at a low level. They could easily build up higher-level knowledge *stepwisely* through discussion with careful supervision. This is unlike traditional knowledge-delivery methods, where high-level knowledge (imparted from teachers) is often inaccessible to students.

Furthermore, students, after engaging in discussion, could more easily memorise the related contents or topics. One notable example can be:

> "Traditionally, when introducing the maximum or minimum point location for a quadratic graph in the general form $\mathbf{y} = a\,\mathbf{x}^2 + b\,\mathbf{x} + c$ ($a \neq 0$), students are passively 'fed' with the notions of its vertex, the sign of a which affects its opening direction and then with some tedious working steps of finding out the exact location of the maximum or minimum point concerned. In fact, learning will be very difficult for those less able students who are weak in mastering such methods of merging the algebraic and geometrical properties of quadratic curves."

On the contrary, in my previous discussion-based lessons, students in groups were asked to freely explore the features of a quadratic curve without applying the merging method. At the beginning, they were stimulated to visualise quadratic curves by raising daily-life examples. Secondly, they were motivated to discover the presence of a vertex of a quadratic curve and

its related characteristics. By giving some guidance like making comparisons of the co-ordinates of each point lying on the curve, group members who were weak in algebra could easily discover the least or greatest value of the y-coordinate for the vertex when the graph is sloping upwards or downwards. Those being even less competent in co-ordinate geometry could grasp the intrinsic character of a maximum or minimum point location when they were urged to give real-life situations, like standing on the top of a conical hill or lying at the bottom of a parabolic-shaped valley. Next, they were given some arbitrary values of a , b and c and were urged to find any specific relationship between such values and the shape of the curve by discussion. Finally, for each maximum or minimum point, they were asked to find its co-ordinates and then figure out any relation between such co-ordinates and the values of a, b and c. Throughout such process, the relevant helpful method of completing the square or other concepts like the discriminant (b^2 - $4ac$) were given.

It should be noted that throughout the learning process, all the words or terminologies involved in the tasks were formulated in students' own words or perspectives without introducing any new, vague key concepts at the very start. According to their own knowledge base at whatever levels, they were expected to construct new knowledge step by step.

Throughout the whole year, I found writings in the student-centred logbook and informal interviews not only served as useful indicators of students' learning processes but also acted as a reliable guide for me to improve any previous teaching method and reset goals for students' learning. For example when teaching several methods of solving quadratic equations, the first method of simple factorisation was nearly omitted as I presupposed that my students had already got sufficient training or understanding in the method since Secondary 3. Yet one student wrote in the logbook:

> "Hardly have I found any discussion or drilling exercise(s) on how to apply the method. Please slow down and stop teaching other methods!!! Let us have another chance to relearn it."

Soon I thought that they urgently needed to get more practice in utilising the method in different situations or even relearn it in various contexts. Consequently, their resulting learning processes were smoothened after such adjustment.

Moreover, based on informal interviews between students and me, I realised that the teaching schedule should be altered from time to time. Sometimes I had to hurry to go through some unimportant boring topics like rates or proportions whilst I had to slow down their learning processes

where abstract conceptual topics, like several types of variation cases and their daily-like applications, were taught.

Another advantage of using the logbook-writings was that sometimes after I made some prompt responses to students' questions, feelings or comments found in logbooks or interviews, they tended to think that I really took good care of them. Therefore, the interaction between students and me could be strengthened as well.

What I have tried to do here is to give a description of some of the "highs and lows" of introducing discussion to my previous teaching. (*) I deeply believe in the value of discussion-based learning and that it can be extended to all grades (from Secondary 1 to 7) in secondary schools. Indeed it was not always the easiest approach for me to use in the past school year, but I am very convinced that it was worthwhile. I will enrich my mathematics teaching experience from year-to-year comparisons when I continue to undertake this action research project.

Reference:

Hustler, D. (Ed.) (1986). ***Action Research in Classrooms and Schools.*** London: Allen & Unwin.

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