Guessing Games in Primary Mathematics

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Introduction

One way to make mathematics more interesting, interactive and enjoyable, especially in the Primary stage, is to cast an activity in the form of a game or puzzle. The type of game/puzzle centred around "Guess what I'm thinking of" provides a simple example of this process and variants of the game in different contexts are easily designed. This article describes four such activities suitable for the Primary age-range and discusses some of their pedagogical benefits. All of these games can be instigated first by the teacher, with the pupils trying to guess the 'unknown', but can also be played with a pupil leading the activity. In the latter case, to safeguard the integrity of the activity, the pupil should tell the teacher what he/she is thinking of so that the teacher can monitor the subsequent answers given by the pupil to the rest of the class. In all the cases described below, after clarifying the 'rules' of the game, the pupils should initially be allowed to play the game freely using whatever strategies they can think of. After a number of examples have been tried, the teacher should ask the pupils to describe their strategies and then, as a class, try to identify the 'best' procedure.

Guess My Number (1)

The teacher thinks of any number from 1 to 100 say, and the pupils try to guess the number by asking questions. The questions can only be of the form that requires a Yes/No answer. The game can be played using only the restricted strategy of 'greater than' or 'less than' questions, or in a more 'open' form where any question can be asked. Both have their value and should be used. In the first case, it is usually not long before pupils identify the 'halving' strategy as being the most efficient. This can then lead to a small investigation based on this idea.The investigation can be developed through a series of prompts: How many questions did we need to find the answer? Was it always the same number of questions? What about for the range 1 to 50? Range 1 to 40? Is it the same? What do we do if we cannot find half the number exactly? Which numbers *can* we keep halving exactly until we get to 1? etc. Although the formal concept of powers of 2 may be considered outside the primary curriculum, this activity is well within the capabilities of most upper Primary school children. In fact, the natural way of expressing their conclusions might be in the form of a table such as:

Largest Number	8	16	32	64	128	....	....
No. of questions needed (before we are SURE of the answer)	3	4	5	6	7	....	....

Such a table tells us that the maximum number of questions needed for a number from 1 to 100 is 7. (This also needs discussion). The pupils can go on to forecast the number required for larger ranges. The power of the method is strikingly illustrated when they see that only 10 questions are required for a range from 1 to 1000. An interesting psychological aside may be mentioned here. When I have played this game with young children I have often found that they are disappointed if the answer to a question is No rather than Yes! (John Holt (1969) described a similar phenomenon in his classic book *How Children Fail*). One of the things that should be discussed at an early stage is that when a halving strategy is used the information gained from a No answer is of exactly the same value as from a Yes answer. Of course, it is precisely the *symmetry* of the halving strategy that makes this so and highlights its advantage.

In the more 'open' form of the game we impose a restriction on the 'greater/less than' questions. That is, only one such question may be asked (or one of each). This now demands more 'creativity' on the part of the pupils in devising further questions. It is quickly realised that one other question also has the power of halving the possibilities: Is it even (or odd)? After that, the answers will not, in general, provide quite as much information. Possible questions are: Is it a multiple of 3 (or 4 or 5 etc)? Is it prime? Is it a square number? Is it a 2-digit number? Is the sum of the digits even/odd? etc. This particular form of the game provides a good opportunity for the pupils to use their conceptual knowledge of a range of properties of numbers.

Guess My Number (2)

This game can be seen as a way of introducing simple linear equations. Again, the purpose is to guess an unknown number and initially is best played as a purely mental exercise. The teacher states the final outcome after a sequence of operations on the unknown number. At first, only two operations should be used, or even just one operation. For example: I'm thinking of a number. If I double it and add 3 the answer is 21. What is the number? Any combination of addition/subtraction with multiplication/division could be used, in any order. Another example: If I subtract 5 and then divide by 3 the answer is 7. Such sequences can be extended to three or even four operations and the solution process is still elementary. The only difficulty lies in remembering the sequence, so now the pupils should be encouraged to jot down the essential information (e.g. +2, ×3, −5; get 19 etc.). The conditions for constructing the sequence are that each operation should be 'well-defined' and that the final numerical value is given. In this context, 'well-defined' refers to any simple numerical operation. Thus, introducing the unknown again into the sequence would *not* count as well defined. For example, the following would not be well-defined: If I add 3, multiply by 4 and then subtract the original number, I get 27.

As in the first game above, after plenty of specific examples have been tackled, there should be some discussion to identify exactly *how* the pupils are finding the answers. It will be clear that some form of 'reversing' or 'unpacking' process is being used. That is, a sequence of inverse operations. At this stage we could represent the method pictorially in the form of a 'flow diagram'. For example:

$$\boxed{?} \xrightarrow[\times 2]{} \boxed{} \xrightarrow[+3]{} \boxed{19}$$

$$\boxed{8} \xleftarrow[\div 2]{} \boxed{16} \xleftarrow[-3]{} \boxed{19}$$

As mentioned before, there is no extra conceptual difficulty in extending this to 4 steps:

$$\boxed{?} \xrightarrow[+1]{} \boxed{} \xrightarrow[\times 3]{} \boxed{} \xrightarrow[-4]{} \boxed{} \xrightarrow[\div 2]{} \boxed{10}$$

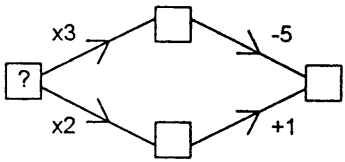
$$\boxed{7} \xleftarrow[-1]{} \boxed{8} \xleftarrow[\div 3]{} \boxed{24} \xleftarrow[+4]{} \boxed{20} \xleftarrow[\times 2]{} \boxed{10}$$

In essence, the pupils have already been solving a set of simple linear equations. If we express the previous flow diagrams in the 'traditional' algebraic form, they become:

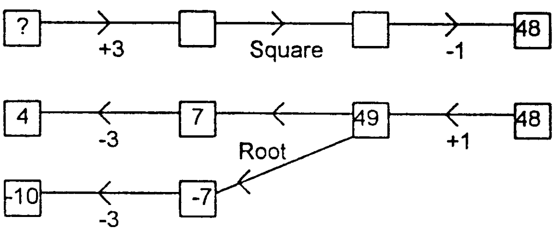
$$2x+3=19 \quad \text{and} \quad \frac{3(x+1)-4}{2}=10$$

Certainly the second equation is far more complex than we would wish to introduce in the primary curriculum, but notice that its complexity is more 'apparent' than real and is effectively a consequence of the symbolic representation. As a flow diagram it is merely a longer sequence than the first example. In terms of teaching the traditional algebraic solution to such equations we need to ask the question: Why should we move to a more complex symbolic method if the flow diagram approach is so simple and effective? In other words, we need a compelling *reason* for doing so. This is where the initial restriction on having a *numerical* result becomes important.

Remember that we have been dealing with a special type of linear equation up to this point. Consider the following problem:
I'm thinking of a number. If I multiply it by 3 and subtract 5, I get the same answer as when I double it and add 1.
Now a flow diagram of this problem is:



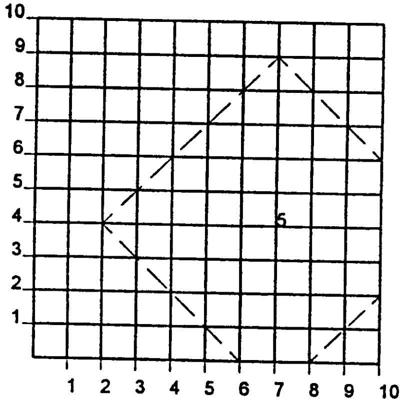
It is clear that applying a sequence of inverse operations is impossible since we have no final number to work back from. It is now the *equality* of the two sequences that is the most relevant feature of the problem. This forces us to look back at our previous problems (that were easily solved by flow diagram) to see whether there is an alternative way to solve them. When we express the first problem as the equation $2x+3=19$ we can match the solution steps with our previous inverse operations so that the pupils see the logic of the process. The important point, from a pedagogical perspective, is that the emphasis is now on the *equivalence* of both sides of the equation (and transformations that retain this equivalence) rather than a sequence of operations.
Although we are concerned here with primary mathematics, it is interesting to note that this approach could also be used to illuminate aspects of quadratic equations at the secondary stage. For example:
I'm thinking of a number. If I add 3, then square it and then subtract 1, I get 48.
As a flow diagram, we have:



Algebraically, this would be: $(x+3)^2-1=48$ and of course it is easily solved using precisely the steps of the flow diagram. However, the same equation could also be expressed as: $x^2+6x=40$. As a teaching strategy, this could lead directly to the idea of re-arranging the equation in a form that can be expressed as a sequence of simple operations. Effectively, this is the rationale behind the method of completing the square.

Guess My Position

We now turn to the spatial context of guessing a position in the co-ordinate plane. Variations of this game have appeared in computer software format and it can also be set in the context of finding 'buried treasure'. However, the basic structure is as follows. Only positive co-ordinates are used (although this could be extended at a later stage) and a suitable size for primary pupils is a 10 x 10 grid. The teacher thinks of a point on the grid and the pupils try to guess its co-ordinates. At each guess the teacher informs the class how far away the guess is from the 'treasure' and this information is placed on the grid. This is illustrated in the diagram below where the guess of (7,4) is shown as 5 units from the treasure. The distance quoted is the *shortest* distance along the grid lines.

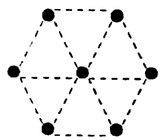


As in the previous games, pupils should try a range of examples first without any hints from the teacher and later the strategies being used should be discussed. Clearly, the first guess is always pure chance (although there can still be discussion on whether choosing the centre of the grid, for instance, is more helpful than other points). However, the information given after the first guess will now determine a smaller set of possible points. This is illustrated by the dotted lines in the diagram. Subsequent guesses, if made logically, will quickly converge to the solution by considering the intersection of the sets of possible points at each stage. This is a simple and enjoyable game that re-inforces the concept of co-ordinates and at the same time encourages a logical thinking strategy in a problem solving situation.

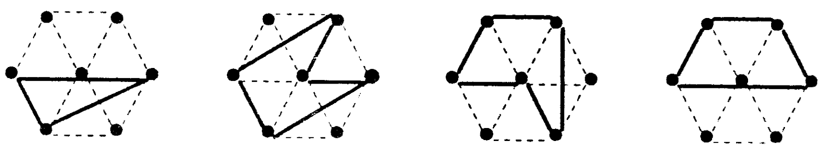
Guess My Shape

The fourth example is concerned with a purely geometric context but has a similar pedagogic function to the first number game. It can be played with many different sets of geometric elements. The illustration given here was first described by Kirkby (1986). The game can be used as an extension or exploitation of the following investigation:

Given seven pins on a triangular grid, as shown in the diagram below, how many different polygons can be formed by joining any number of pins? (In practice, the activity can be done on a pinboard with elastic bands).

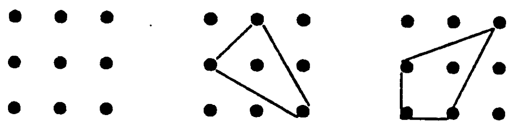


Four possible polygons are shown in the diagrams below. In all, 19 different polygons can be drawn. (Precisely what is meant by *different* also needs to be discussed. Here it is taken to mean non-congruent). It is left for the reader to find the remaining possibilities.



The investigation itself raises questions like: How can we be sure we have found all the possibilities? How do we use a systematic approach to ensure we do not miss or repeat any shapes? However, it is the final outcome we are interested in here. Once all the polygons have been found a large poster can be drawn illustrating them all together and labelling each by a letter or number. This can then be used as the basis for a similar game to those previously discussed. The teacher thinks of one of the shapes and the pupils must guess which one. As in the first activity, only questions that can have a Yes/No answer are permitted. Again, the pupils need to be creative and to use their conceptual knowledge of the properties of shapes in order to think of a variety of suitable questions. Some possibilities are: Does it have more than 4 sides? Does it have any right angles? Any obtuse angles? Does it have any symmetry? More than one line of symmetry? Does it occupy less than half the area of the grid?

Clearly this game can be played with any set of geometric shapes. Another simple investigation that would produce such a set could come from a 9-pin rectangular grid:



The total number of possibilities here is far more numerous than before so it is advisable to restrict the investigation first to the number of different triangles that can be found and then the number of different quadrilaterals. Again, a poster showing all the possibilities for these two cases can form the basis of the game. (One example of each is shown above. The reader is encouraged to find the remaining shapes. There are 8 triangles and 16 quadrilaterals).
Of course, it is not necessary to generate the shapes from an investigation, as described here. *Any* suitable set of polygons will do. Also, the activity need not be limited to plane shapes; it can just as easily, and usefully, be played with a set of solids.

Discussion
By now, most Primary teachers in Hong Kong are very familiar with the five ways of learning and using knowledge recommended in the TOC Programmes of Study documents. That is: Communicating, Conceptualising, Inquiring, Problem-solving and Reasoning. Such ways of learning should not be seen as dichotomised and it is almost inevitable that any learning situation will involve more than one of these elements in an integrated manner. Nevertheless, it may be that in particular cases one may dominate more than another. How well do the activities described above incorporate these elements? There are two main aspects that feature throughout, namely problem-solving and communicating. The games themselves are essentially of a problem-solving nature since the *objective* in each case is to determine an unknown. But the structure of the games is heavily oriented towards communication and it is the pupils themselves who are placed in the position of asking the questions. This is a reversal of the usual state of affairs in a classroom where the teacher appropriates most of the questionning that takes place. These simple games thus offer an opportunity for the pupils to express themselves mathematically and to be precise in their language. This is a much neglected feature of mathematics teaching.
The other three ways of learning are also evident in these activities. The kind of questions needed for the first and fourth games certainly involve understanding and using the properties of number and shape and will

strengthen the pupils' conceptual knowledge in those areas. Equally, the second game will help with the conceptualisation of the process of solving an equation while the third reinforces the concept of co-ordinates. The investigation that arises from the first game and those that can introduce the Shape game are very clear examples of Inquiring. And finally, all the games need an element of reasoning in determining the unknown but this aspect is particularly evident in the Position game. It is hoped that the guessing games illustrated in this article can provide some helpful ideas for Primary teachers and also serve as a stimulus for devising their own games based on the same principles.

References

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