Developing Mathematics Learning Materials Using Freeware

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Abstract

The Hong Kong Special Administrative Government (HKSAR) has been encouraging the teaching professionals at all levels to apply Information Technology (IT) in education. With a substantial amount of public money being poured to support this direction, many research and development projects have been completed or are undergoing to improve the quality of teaching and learning (T&L) of mathematics, one of the major disciplines in both primary and secondary curricula. In general, a mathematics teacher can obtain some fixed IT packages through three channels: free products sponsored by the government or other organizations such as tertiary institutes and book publishers, commercial packages in the market, and free package downloaded from the Internet. Some of these ready packages are very useful to teachers for their teaching and students for their learning. However, there is one major shortcoming among these fixed packages: they do not allow the flexibility for a teacher to develop school-based teaching materials. In order to develop dynamic and interactive programs for their own students, schools may need to purchase license(s) of some expensive software. For schools that do not have enough resources for buying software, using freeware is another alternative. In this paper, a freeware called "WinGeom" is introduced. In addition, three examples for the T&L of mathematics developed in the platform of WinGeom are described and elaborated.

Making Use of Freeware

As practitioners of mathematics education, the effectiveness of teaching and learning of mathematics is our major concern. Research findings in the past decade indicated that Computer-Mediated Instruction (CMI) has been found to have moderate effect on the reinforcement of student achievement (Kulik, 1991; Vacc, 1992). Many local educators are also aware that implementing CMI in teaching will be beneficial to our students. Shortage of hardware equipment is no longer a problem as the HKSAR has promised to allocate sufficient number of multimedia computers to schools. However, the scarcity of CMI materials with local context (Math. Section, 1996) related to mathematics remains a major concern of the teaching professionals.

If a mathematics teacher wishes to use IT in classroom teaching, she or he may obtain some fixed packages through three popular channels: (i) free products developed and sponsored by the government, tertiary institutes, and/or book publishers (ii) commercial packages in the market, and (iii) free packages downloaded from the Internet. Some of these ready packages are very useful to teachers for their teaching and students for their learning. However, there is one major shortcoming among these fixed packages: they do not allow the flexibility for a teacher to develop school-based teaching materials. Some interactive programs have been developed by local researchers to illustrate how to make use of free platforms such as spreadsheet for developing self-designed materials (e.g. see Kong, Leung & Leung, 1997; Kong, Man & Leung, 1999; Leung, Kong, & Man, 1998; Leung & Man; 1998, Man & Leung, 1998). There are many advantages of developing interactive T&L packages using spreadsheet such as: (i) it is a generic software that many teachers are experienced in using it, (ii) it supports the Chinese characters, and (iii) it does not cost extra money as it is a standard software accompanying with the hardware delivered to schools. Nevertheless, spreadsheet is not specifically designed for the T&L of geometric topics that form essential parts of mathematics curricula. In order to develop dynamic and interactive programs for the T&L of geometry for their own students, schools may need to purchase license of some expensive software. For schools that do not have enough resources for buying software, we suggest an alternative of using freeware. In this paper, we wish to introduce the freeware of WinGeom that can be downloaded from http://math.exeter.edu/rparris/. The advantages include: (i) it is specifically designed for the T&L of geometry, (ii) there is no worry for schools to raise funding for the license, (iii) it is easily accessible by the users, and thus (iv) it is anticipated that many programs will be developed on this platform and shared among mathematics educators. In the following section, three examples for the T&L of geometry developed in the platform of WinGeom will be described and elaborated.

Examples

<u>Example 1</u> Angle Sum of Triangle

One of the traditional way for teaching the topic of angle sum of triangles is by paper folding: cut a piece of paper into a triangle in any shape and fold it into a rectangle as illustrated in Figure 1.

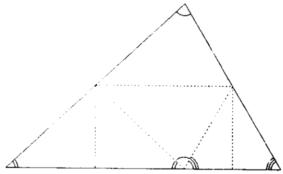

Figure 1

Figure 1 shows that the three angles of the triangle are supplementary angles of a straight line and form two right angles. Thus, the angle sum of that triangle is 180°.

Using WinGeom, we can construct a triangle at random and ask the computer to calculate the sum of the three angles. When the shape of the triangle is changed, the program will immediately update the new sum. Figure 2 are two screen snaps of the constructions.

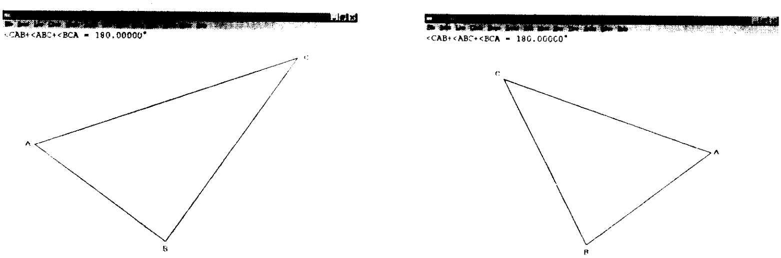

Figure 2

Students can explore and consolidate the concept by generating as many examples as they wish simply by dragging the mouse. In addition, teacher may raise students' interest by carrying out an experiment that cannot be done in traditional paper folding. The task of the experiment is to reduce the size of a triangle gradually (refer to Figure 3). By doing so, students will know that the program may lose precision in some circumstances. No matter how accurate the measurement and computation the computer offers, it only helps us to discover some "potential" facts that we have to provide rigorous mathematical proofs for their generalization.

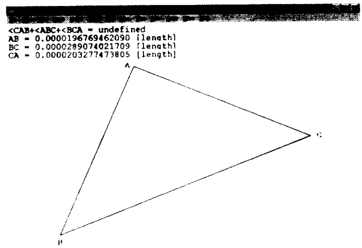

Figure 3

<u>Example 2</u> Approximation of π

By measuring the circumference and diameter of circular objects, students will have the idea that the ratio between these two attributes remains virtually unchanged and has a value around 3. Teachers can use WinGeom to carry out further investigations and lead students to the conclusion that this value cannot be represented "easily". Thus, so we better use a symbol, conventionally π, to represent it. We start by asking the computer to generate a regular polygon with side length of 1 and let its perimeter be an approximation to the circumference of the circumcircle. Figure 4 illustrates the case of a 7-gon with its circumcircle.

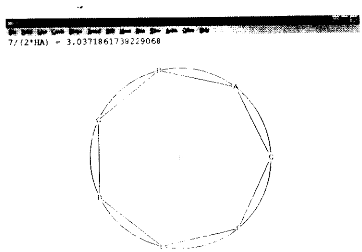

Figure 4

Notice that, since the side length of the 7-gon is 1, so 7/(2×HA) gives an approximation to the desired ratio. Moreover, it is clear from the diagram that the circumference of the circle is greater than the perimeter of the 7-gon, the value of the desired ratio should be strictly larger than 7/(2×HA). In order to get a better estimate of the desired ratio, the students can increase the number of sides of the regular polygon and then observe how the estimate changes. This process can be continued until a very "good" approximation, as agreed by both teacher and students, is reached.

Figure 5 illustrate the case of a regular 57-gon and its circumcircle. They are so close that it is difficult to distinguish them.

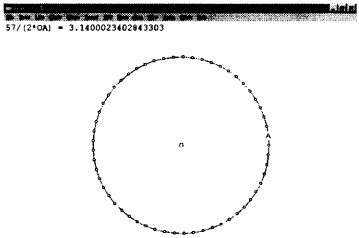

Figure 5

The teacher, after doing the construction together with the students, can lead students to the following conclusions: (i) 3.14 is a "good" approximation of π, the desired ratio, and hence can be used in daily computation, (ii) the exact value of the desired ratio can not be written down easily and people agree to use π to represent it, and (iii) π is strictly bigger than 3.14.

Notice that the fraction, 22/7, is also a commonly used approximate of π. But $22/7 \approx 3.1429$ which is bigger than 3.14. To let students realize that 22/7 is also an appropriate estimated value of π, teachers can use the incircle of a regular polygon instead of circumcircle. Figure 6 illustrates the case for a 7-gon. It is convincing that the length of the perimeter of the 7-gon is bigger than the length of the circumference of the incircle. Now, as before, we have an approximation to π but this time $7\times(2\times OH)$ is bigger than π.

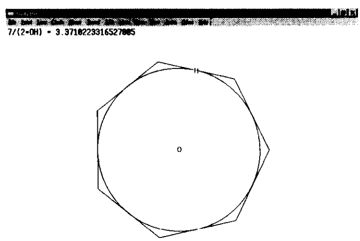

Figure 6

Figure 7 illustrates the case of a 91-gon. The conclusion now is that 22/7 is a very "good" approximation of π and is bigger than π.

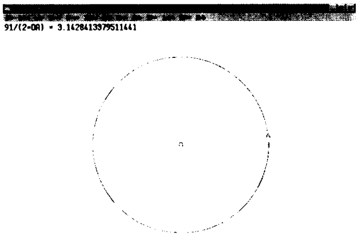

Figure 7

Example 3 Isoperimetric Triangles

Suppose students have already learnt the formula for calculating the area of a triangle. They can perform the following investigations that require students to use the formula with higher order thinking.

First, construct a triangle with perimeter of constant length 20 and then measure the lengths of the sides and its area (refer to Figure 8).

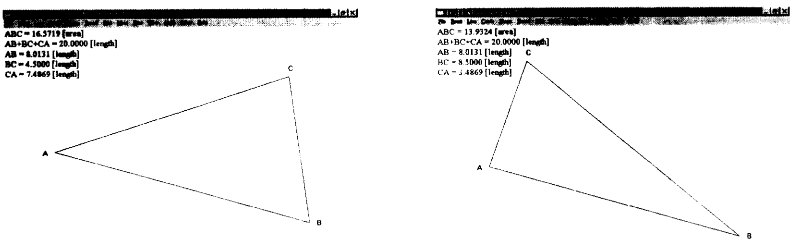

Figure 8

Students are asked to move the point C and observe when the area will be maximal. They should be able to observe that the area becomes larger when the triangle is closer to an isosceles triangle. After the exploration, students are invited to explain their findings by using the formula for the area of triangle. Figure 9 shows the trace of the point C as it moves. Since the area of a triangle is given by half of its base times height, students should realize that on a fixed base the area is maximum when the height is largest. This occurs exactly when the point C is "highest" above AB. In such circumstance, the triangle is "symmetric" (or isosceles).

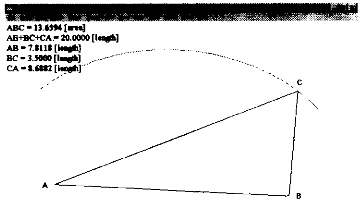

Figure 9

Further investigations and discussions can be carried out if students are encouraged to move all the three points. They will observe that the area would become larger when the triangle approaches an equilateral triangle. With the previous discussion for the isosceles case, students will be able to give an explanation for this finding under guidance.

Discussion

Computer is definitely a very powerful tool for the teaching and learning of some mathematics topics. It cannot work on its own and must be accompanied with software. Freeware is a free resource that provides a platform for researchers and practitioners to develop dynamic and interactive programs according to their needs. In this paper, three examples for the T&L of geometry using the platform of WinGeom are described. By exploring through the examples mentioned, students can develop and consolidate the relevant concepts. Besides, they may develop an appreciation of using IT in learning and realize that IT only offers potential facts that need rigorous proofs for generalization. In addition to introducing the use of popular freeware such as WinGeom, we also wish to promote a culture of mutual sharing among the profession by listing all the programs for the examples in the appendix. If we build up such a culture of sharing self-developed T&L materials, the aggregation will benefit the whole community of mathematics education. We anticipate that more and more well developed teaching programs will be published in the EduMath and shared among the professionals for mathematics education.

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Appendix

The constructions described in the paper can be done on WinGeom (version at 3 Mar. 2000) as follows.

Angle sum of triangle

1. Press **F2** to open a new 2-D construction.
2. Select **Shape:Random:Triangle** from pull down menu to generate a random triangle *ABC*.
3. Select **Meas** and enter <ABC+<BCA+<CAB in the dialog box to calculate angle sum.
4. May select **Others:Measurements:Drawing/printing font ...** to change fonts for displayed text. (The fonts Terminal or MSLineDraw are recommended by the author of the software.)
5. Select **Btns:Drag points** to set the mouse buttons to drag point mode.
6. Drag the vertices of the triangle to observe changes.

Approximation of π

1. Press **F2** to open a new 2-D construction.
2. Select **Shape:Polygon:Regular...** and enter the number of sides to construct a regular polygon. Note that we use the default side length of 1.
3. Select **Circle:Circumcircle...** and enter ABC to draw the circumcircle of triangle *ABC*. Notice that the circumcircle of the triangle *ABC* is the same as the circumcircle of the regular polygon.
4. Select **Meas** and enter the appropriate equation for ratio between the length of the perimeter of the polygon and the diameter.

5. The incircle can be constructed similarly except that now the incircle of the triangle *ABC* is not the incircle of the regular polygon. One way out is to construct the midpoint of a side using **Point:Midpoints...** and construct the incircle by **Circle:Radius-center...** using the circumcenter as center and the distance between the circumcenter and the midpoint constructed as radius.

Isoperimetric triangle

1. Press **F2** to open a new 2-D construction.
2. Select **Btns:Segments** to set the mouse buttons to segment drawing mode.
3. With the mouse cursor in the construction area right click the mouse button to draw point *A* and *B*. Joint the segment by moving the cursor to point *A* and left drag to point *B*.
4. Select **Circle:Radius-Center...** to draw a circle with center *B* and of radius 10x#. Notice that # is a parameter used in WinGeom. Use * to represent × in the dialogue box.
5. Similarly draw a circle with center *A* and radius $20-AB-10*\#$.
6. Select **Anim:\# slide...** and drag the horizontal scroll bar to change the value of #. Make the circles intersect.
7. Mark the intersections of the circles as *E* and *F* by right click on the intersections (with mouse button in segment mode).
8. Select **View:Labels:Swap...** to swap the labels of points *E* to *C*. Join segments *AC*, *BC*.
9. Select **Edit:Circle delete...** to delete the two circles.
10. Select **Edit:Point delete...** to delete points *D*, *E*, *F*.
11. Measure the side length, perimeter and area of the triangle.
12. Shape of triangle can be changed by dragging points *A*, *B* or by changing the value of #.
13. Select **Anim:Tracing...** and choose **New**. Set **control** to # and **pen on** *C* to trace the movement of point *C*.