# The Role and Use of Word Problems in School Mathematics

Yeping Li
Learning Research and Development Center, University of Pittsburgh

One important change in school mathematics in many nations has been suggested to shift the instructional emphasis from students' acquisition of procedural proficiency to the development of their problem-solving ability (e.g., National Council of Teachers of Mathematics [NCTM], 1980, 1989; Zhang, Sawada, & Becker, 1993). Such suggested change has consequently raised various concerns and generated broad discussion about the types of math problems that students are expected to be able to solve, and how math problems should be used in teaching and learning mathematics in classroom. One of the important concerns has been on the role and use of "word problems" in school mathematics. Certain questions related to word problems are worth pursuing in the international context of school mathematics reform movement. They included "Does solving word problems serve a better function than solving purely mathematical problems to foster students' problem-solving ability?" and "How can word problems be adopted in math curriculum and classroom instruction as problem contexts to facilitate students' learning of school mathematics?" To pursue this type of questions, this paper takes a close look at word problems that were placed in school mathematics in imperial and modern China and Chinese students' mathematical performance in solving different word problems, with possible references to the case of the US. Such discussion is expected to lead us to a further understanding about the role and use of word problems for improving mathematics instruction in China.

*Word Problems in Ancient Chinese Mathematics and School Mathematics*

The idea of adopting word problems for introducing mathematics is not new in China. As a matter of fact, presenting and solving word problems were the main approach used in the development of ancient Chinese mathematics. This approach resulted in an accordance between ancient Chinese mathematics and its practical nature. For example, the well-known book, Nine Chapters on the Mathematical Art (Jiuzhang suanshu, 25-220 AD), was specifically about mathematics and was compiled with various word problems from the real world and the rules used to obtain their solutions. There are a total of 246 word problems in this book that were organized into nine chapters with specific titles. For example, Chapter one was named "Fang tian" (Field measurement), and Chapter three was called "Cui fen" (Distribution by proportion). The following is a typical word problem concerning "series" included in Chapter three (Swetz, 1972, p.425):

A girl skillful in weaving doubles the previous day's output of cloth. She produces five feet of cloth in five days. What is the result of the first day, and successive days, respectively

The presentation of word problems was followed by their solutions. The form of problem-and-solution used in this book is essentially an inductive approach and historically had dramatic influence on the development of ancient Chinese mathematics (Li & Du, 1987).

Although ancient Chinese mathematics had tremendous development with the tradition of solving word problems, the form and requirement of mathematics in formal schooling and civil examination in ancient China were totally different from the prominent achievement and practical nature of ancient Chinese mathematics. Specifically, a dramatic concentration was placed on literature in civil examination and formal schooling in ancient China. Practical knowledge and skills like arithmetic were treated as incompatible for fostering scholars and selecting state officials with moral superiority (Hu, 1984). Moreover, with the influence of Confucian's thought that Saints' books1 were all right, rote learning of Saints' books became a popular approach in ancient Chinese education. The approach of rote learning was further reinforced by the requirements of civil examination in ancient China. Even when mathematics was required to learn in ancient China, the approach used in learning mathematics was also rote learning. Therefore, school mathematics in ancient China was an unimportant subject and had no intention to foster students' problem-solving ability.

With the influence of Western mathematics and the development of new schools at the beginning of this century, school mathematics became an important subject and began to adopt the format and requirements from Western mathematics. As described by Li & Du (1987), "the textbooks used in the various kinds of Western school (in China) were changed again and again until finally there was essentially no difference between them and the mathematical textbooks used all over the world" (p. 256, italic added). Therefore, the tradition of problem-and-solution approach developed in ancient Chinese mathematics was unfortunately separated from the approach used in school mathematics in ancient China.

*Word Problems in Current School Mathematics Curriculum*

The role of word problems has been emphasized in current school mathematics curricula for all grade levels. Specifically, it is stated as one educational purpose in the national syllabus in the People's Republic of China (PRC) that students are expected "... being able to apply knowledge to solve simple real-world problems" (for Grades 1-6, Grades 7-9), and "... being able to apply mathematical knowledge to analyze and solve real-world problems" (for Grades 10-12). Such statements

indicate expected students' mathematical competency that relates to word problems. However, it is unclear from such general statements how word problems are adopted for introducing mathematics as well as how word problems are presented in textbooks for students' practices.

Two recent studies on mathematical textbooks provide detailed information about Chinese textbook content presentation and problem characteristics (Carter, Li, & Ferrucci, 1997; Li, 1998a), which allows me to take a closer look at the role of word problems in current mathematics curriculum. In the first study on content presentation and organization in sections on integer addition and subtraction in four Chinese textbooks, it was found that the way of introducing the notion of integer addition in these Chinese mathematical texts was using a word problem (Carter et al., 1997). The word problem was used as a context to provide a concrete analogy of adding integers and was solved with the coordinating uses of verbal and symbolic representations (and possibly pictorial representation in some texts). This is an approach that was not common in several US texts being analyzed. Moreover, all of these Chinese texts presented "integer addition and subtraction" as a subspace of the rational number operations and used an inductive approach to develop general addition rules, which showed higher math requirement than the US texts. Therefore, this snapshot analysis indicated that Chinese textbooks tried to adopt word problems for introducing mathematical content. This is an instructional approach that connects instructional content not only with a problem context but also with more advanced study in mathematics.

In the second study on problem sets in sections on integer addition and subtraction in the same four Chinese textbooks, Li (1998a) examined expected students' math experience that was envisioned in textbook problem sets. Specifically, the majority of math problems presented in these content sections in these Chinese and several American texts were purely mathematical problems (90% in Chinese texts vs. 87% in American texts). Although only about 10% of the problems in Chinese texts were word problems, there were only slight differences between Chinese and American texts in terms of the percentages of purely mathematical problems and word problems that were analyzed in corresponding sections on integer addition and subtraction. However, a fine-grained analysis of mathematical problems conducted in this study showed that there were dramatic differences in mathematics problems between the American and Chinese texts. Such dramatic differences were found not in the form of mathematical problems but in the problem requirements. For example, problems classified as requiring a mathematical solution constituted 13% in the US texts but none of those in the PRC texts. Likewise, about 6% of US textbook problems required verbal explanation but no such requirement presented in Chinese textbook problems. Several sample problems taken from the US texts are given below (see Figure 1). Because the problems that required a mathematical solution also asked for a mathematical (e.g.,

pictorial) explanation of the solution process, these explanatory differences (13% and 6%) evidenced in the US and Chinese texts illuminated the cross-national differences in the educational expectations that were envisioned in the textbook problems.

**Figure 1.** Several mathematical problems presented in the US middle school math textbooks

Problem 1 (a word problem that asked students to provide verbal explanations)
In an offensive drive in the football game, Troy's team lost 6 yards, lost 5 yards, gained 14 yards, and then gained 7 yards. If the team has to gain 10 yards in four plays to make a first down, did Troy's team make a first down? Explain.

Problem 2 (a purely mathematical problem that asked students to provide a solution with a pictorial representation)
Use a number line to find the difference: -21 - (-10)

Problem 3 (a purely mathematical problem that asked students to pose a word problem that can be represented with a given addition)
Describe a situation that can be represented by the addition -15 + (-45)

The textbook problem comparison indicated that the US texts included more variety of problem requirements than Chinese texts, a result that was not evident in the first study on content presentation. Although Chinese texts presented an interesting approach of introducing "integer addition and subtraction" using a word problem as context, Chinese textbook problems presented for students' practices showed no specific advantage (except higher math requirements) in improving students' problem-solving ability as compared to the US texts. In fact, the differences embedded in how math problems were required to solve indicated that Chinese students lacked relevant opportunities of solving problems with diverse requirements, though there were no dramatic differences in the percentages of word problems being adopted in problem sets between the US and Chinese texts. Taking together, these studies indicate that the role of word problems in school mathematics needs to be explored beyond whether word problems were included in texts or classroom instruction. More importantly, how word problems are presented and required in teaching and learning mathematics can inform us how well the word problems may serve for the purpose of fulfilling a nation's educational expectations.

*Chinese Students' Mathematical Performance in Solving Different Word Problems*

Above discussion has focused on how the role of word problems was played out in Chinese mathematics textbooks. To understand how well the word problems in textbooks might serve for the purpose of fulfilling a nation's educational expectations, it is imperative to know how well Chinese students are capable to solve word problems.

Several cross-national studies have consistently shown that Chinese students had good performance in solving traditional math problems, such as numerical computations, but were not similarly good in solving open-ended or novel problems. For example, the IAEP (International Assessment of Educational Progress) study conducted by ETS (Educational Testing Service) in 1991 showed that Chinese students had the best mathematical performance among thirteen participating nations and regions in solving computational tasks and simple word problems (Lapointe et al., 1992). Likewise, Stevenson and Stigler (1992) reported that fifth graders sampled in Beijing had better performance than their American counterparts sampled in Minneapolis in solving both computational items and (simple) word problems. However, Chinese students' mathematical performance was found to be unexpectedly poor in some other studies. For example, Zhang (1992) reported that a group of ninth graders and a group of twelfth graders at an ordinary high school in Yellow Mountain city had dramatically low average scores, 6.76 and 7.2 out of 40 points respectively, in solving four open-ended mathematical problems that were selected from four foreign math textbooks (see two sample problems in Figure 2 below). According to Zhang (1992), those Chinese students did not understand what they were asked to do with these "foreign" problems.

**Figure 2.** Two of the problems used to test Chinese high school students (Zhang, 1992)

Problem 1 (from Britain, 10 points)
A piece of paper can be torn into five smaller pieces. Each of the small pieces can be torn again or not. Continue as you like. Can you get exactly 1,989 pieces at a single moment?

Problem 2 (from U.S.A., 10 points)
Nine robots are to perform various tasks at fixed positions along an assembly line. Each must obtain parts from a single supply bin to be located at some point along the line. Where should the bin be located so that the total distance traveled by all the robots is minimal?

Similarly, Cai & Silver (1995) reported that Chinese students had tremendous successes in division computations but not in solving the following Division-With-Remainder (DWR) word problem as compared to their American counterparts .

Students and teachers at Guangming Elementary School will go by bus for spring sightseeing. There is a total of 1128 students and teachers. Each bus holds 36 people. How many buses are needed?

Because a successful solution to the DWR problem required students' interpretations of a remainder in terms of the problem requirement, this is a solution requirement that needs students' sense-making and reasoning and is different from numerical computations. These results indicated that Chinese students may have developed certain mathematical competency to a high degree but not a super ability for solving different word problems, especially the ones that have different problem requirements or are novel to them. Then, what have Chinese students developed through their learning of school mathematics? In light of my recent investigation on Chinese students' mathematical competency in solving different algebraic problems (Li, 1998b), Chinese students have seemingly developed a strong mathematical pattern that is operationalized and rooted in purely mathematical content.

Chinese students' performance in these studies might suggest that word problems in Chinese mathematical textbooks have been efficient in fulfilling some educational purposes but may not be good enough in fostering students' ability of solving real-world problems. Specifically, the findings on the characteristics of mathematical problems presented in previous analyses on Chinese texts have provided partial explanation about Chinese students' weaknesses in solving mathematical problems with novel problem requirements. Therefore, it is left for us to understand how the adoption of word problems may be improved in Chinese school mathematics for fulfilling its educational expectations.

*Rethink the Role and Use of Word Problems in Today's School Mathematics*

Although ancient Chinese mathematics has the history of adopting word problems as a means for developing and presenting mathematics, such feature, unfortunately, was not evident in school mathematics. In fact, some educators have argued about the feasibility of adopting the problems from ancient Chinese mathematics for teaching and learning mathematics in Today's schools (e.g., Swetz, 1984, 1995). The examples they presented did illustrate their potential of promoting students' interest in learning mathematics and of developing their mathematical thinking in today's classrooms.

However, it is not easy to determine whether the use of word problems in school mathematics can serve expected functions. The purpose of adopting word problems is to serve educational needs in today's school mathematics rather than in adoption

itself. Even with the ready available word problems in ancient Chinese mathematics, we need to think about why and how to adopt word problems in teaching and learning school mathematics. Specifically, the following are several questions that may need to be considered in order to improve the use of word problems in Today's school mathematics.

Why do we need to adopt word problems in Today's school mathematics?

As mentioned at the beginning of the paper, the suggestions of adopting word problems in Today's school mathematics were generated basically with two reasons: (1) fostering students' problem-solving ability, and (2) teaching mathematics effectively with the use of word problems. Although these two reasons are all related to the use of word problems, the first reason indicates that solving problems is the purpose of mathematics instruction, whereas solving problems serves as a means for mathematical instruction in the second. Because the second reason relates to the questions for the next section, I will focus on the first reason here.

The first reason is based on the assumption that students' experience in solving word problems in the classrooms can develop their ability of applying mathematical knowledge to solve problems in real world. Specifically, it is generally taken for granted by many educators that (a) purely mathematical problems are different from real-world problems, and (b) word problems are real-world-like problems since word problems can provide certain problem contexts that are absent in purely mathematical problems. Because what students may obtain through solving purely mathematical problems (mainly procedural proficiency) is different from what they need for solving real-world problems (problem-solving ability), it is generally true that purely mathematical problems are different from real-world problems in many ways. However, the relationship between word problems and real-world problems may not be simple as what many people thought to be.

First of all, the context provided by a word problem may not have the expected effects in influencing students' problem-solving performance. For example, the context provided by a word problem may have effects on students' problem-solving performance in Brazil (see Carraher, Carraher, & Schliemann, 1987); but such effects may not be evident in students' problem-solving performance in the US (see Baranes, Perry, & Stigler, 1989). Moreover, existing studies indicated that school problem-solving experience do not always equip students with expected mathematical power (e.g., Carraher, Carraher, & Schliemann, 1987; Lave, 1988). Many studies have documented that school students and adults often use their informal strategies, rather than the formal strategies that were obtained and used in solving word problems in the classrooms, to solve real-world problems (e.g., Lester, 1989; Scribner, 1984). Therefore, students' experience in solving textbook word problems in the classrooms

may not have the expected function in developing their ability of solving problems in real world. As argued by Lave (1993), word problems in the classrooms reflected the microcosm of classroom culture in which word problems are often problematic in teacher's perception but not in students' eyes. Thus, the assumption, that students' experience of solving word problems in the classrooms can develop their ability of solving problems in real world, needs to be validated psychologically and culturally.

How may we adopt word problems in today's school mathematics?

As discussed above, word problems have been adopted in current Chinese mathematics textbooks for introducing mathematical content. Although there may be a lot of variations on how word problems can be used for different instructional purposes (e.g., math problems are given to students for free explorations, math problems with solutions are presented as worked-out examples), Chinese texts have evidently developed an interesting approach for developing students' math understanding through the coordinating uses of different representations for explaining a problem's solution. This is an approach that neither asked students to think by themselves nor simply told students the concepts or rules, but introduced math content within a problem context. Even though such results were restricted on a snapshot analysis of content presentation in sections on integer addition and subtraction in Chinese texts, the results showed that the way of word problems used in content presentation may require less changes than the way of word problems presented for students' practices.

Although students' experience of solving various word problems in the classrooms may not have the expected function of developing students' ability in solving problems in real world, word problems certainly have their special advantages over purely mathematical problems that were dominant in traditional school mathematics. In particular, different word problems can provide various complex contexts that go beyond simple or direct application of math knowledge. Therefore, word problems in Chinese textbooks may require changes, but they still serve as a key role in school mathematics for developing students' problem-solving ability.

In order to develop the use of word problems in textbooks, it is necessary to know the purposes that word problems are capable to serve. According to Zhong, Ding, and Cao (1980), mathematical problems presented in Chinese textbooks could be classified according to the instructional purposes they serve. In particular, there are seven types of mathematical problems in textbooks' problem sets that serve different instructional purposes . They include:

1. The problems that help students understand new concepts, condition, and results of new proposition.
2. The problems that help students be familiar with new formula, rules, and skills.

1, 2 are usually included under the title 'exercise' in textbooks, they are the simplest and basic problems. These problems are a few, and are often presented right after the new knowledge was introduced.

3. Proof problems which ask students to apply new knowledge (concept, proposition, and formula). This type of problems is a few, and is usually included under the title of 'problem'.
4. Proof problems which require students to apply new and previous knowledge together.
5. Computational and drawing problems which require students to apply new and previous knowledge together.

4, 5 are the major part of 'problem' in textbooks.

6. General problems which require students to use many different knowledge. This type of problems is few, and is often included under the title of 'review problem'. It has been suggested that this type of problems needs to be increased due to its potential of improving students' ability.
7. The problems which aim to provide some preparations for the follow-up new knowledge to be introduced. This type of problems is few, and is spread in 'problem'. (p. 81)

It seems that word problems can literally be adopted to serve for all different instructional purposes. However, the format and requirement of word problems may vary as instructional purposes are changed.

For the purpose of developing students' ability of solving problems in real world, I would like to suggest the inclusion of diverse word problems. To be specific, the word problems adopted in school mathematics need to show their variety in a way that mimics the problems faced in real world. For example, textbooks need to include the problems with abundant information, the ones with deficient information, and the ones that can not be solved. More importantly, the word problems adopted in textbooks should be "problematic" in students' eyes and present students a "realistic" challenge, which can be dealt by applying their math knowledge and skills. For example, the following two problems present different challenges to students.

**Figure 3.** Two word problems with similar contexts but different requirements

Problem 1: A dark bag filled with 100 red, white, black, or yellow balls is used in a raffle game. All the balls have the same size. Each drawing costs $2.00. The amounts of different colored balls are listed below along with their prices.

| Ball's color | Amount | Price |
|---|---|---|
| Red ball | 5 | $50.00 |
| Yellow ball | 15 | $20.00 |
| White ball | 25 | $10.00 |
| Black ball | 55 | none |
| Total | 100 | |

John wants to try his lucky in this game. How likely will he get a red ball in one draw? How likely will he get nothing in one draw? If he draws 4 times, how likely will he lose money?

Problem 2: Mary is the chair of a club. The club plans to develop its activities by having a drawing game. One club member made a proposal as given below:

A dark bag filled with 100 red, white, black, or yellow balls is used in a raffle game. All the balls have the same size. Each drawing costs $2.00. The amounts of different colored balls are listed below along with their prices.

| Ball's color | Amount | Price |
|---|---|---|
| Red ball | 5 | $50.00 |
| Yellow ball | 15 | $20.00 |
| White ball | 25 | $10.00 |
| Black ball | 55 | none |
| Total | 100 | |

In order to decide if this proposal can be adopted, Mary needs your help. Can you analyze the proposal and tell Mary your suggestion? Explain your analysis and reasoning. If you think that this proposal needs to be changed, explain how to make change(s) and why.

Although both problems are word problems, the second problem is more open-ended and demanding than the first problem. While getting a mathematical solution is a direct requirement for the first problem, mathematics is expected to be used as a means to reach a conclusion for the second problem. Therefore, the second problem shows a real-world-like challenge that goes beyond getting a purely mathematical answer. Although word problems like the first one are also good to be used for students' problem-solving activities and we can find a few of them in math textbooks, it is very difficult to find word problems like the second one in school curricular materials. It becomes necessary and important for math educators

to reflect on the problems adopted in math textbooks and to think about what changes need to be made in the use of textbook problems.

*Final Remarks*

Although many educators are quite familiar with word problems that are used in teaching and learning school mathematics, it is not a trivial task to discuss the role and use of word problems in school mathematics. Because it is often difficult for us to find possible advantages and disadvantages of what is happening in school mathematics without comparisons, we are not sure what needs to be changed for improving our students' problem-solving ability. However, by standing in an international and historical context, we can understand better about current situation in a nation's school mathematics. On the specific topic about the role and use of word problems in Chinese school mathematics, this paper illuminates the advantages and drawbacks of word problems that were adopted in Chinese math textbooks and reflected in students' performance. Further research efforts are needed to explore what and how word problems are used in teachers' teaching in Chinese classrooms, and to develop the use of word problems in textbooks for improving its potential of fulfilling Chinese educational expectations.

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