Using Euclidea (Interactive Geometric Construction Game) to Facilitate Experiential Learning on Geometry

CHUNG Wing Hong Patrick
Po Leung Kuk Celine Ho Yam Tong College

Abstract

Simple geometric construction is included in junior mathematics curriculum in Hong Kong, but teachers always find that it is challenging to be taught. Euclidea, which is an interactive geometric construction game based on dynamic geometry environment (DGE), shows a great potential to facilitate students' experiential learning on geometry. This paper analysis a case of two secondary 5 students engaging in geometric construction by using Euclidea. Through the lens of DGE and experiential learning theory, it shows the possibility of Euclidea in facilitating and inhibiting students' justification of their geometric construction.

Keywords: Euclidea, geometric construction, dynamic geometry, experiential learning, justification.

Literature Review

(I) Geometric construction in mathematics classrooms

In the junior secondary mathematics curriculum of Hong Kong, students are supposed to learn how to do simple geometric construction (also known as straightedge and compass construction) such as equilateral triangle, square, regular hexagon, parallel lines, perpendicular lines, angle bisector and perpendicular bisector (Curriculum Development Council, 2017). However, Hung (2014) states that teachers may probably skip the teaching and learning activities about geometric construction in practice. One of the reasons is that geometric construction is not included in public examinations nowadays (Questions related to geometric construction were only included in HKCEE during 1950s to 1970s). Also Hung points out that some teachers may not be familiar with geometric construction and find it difficult to teach. Teaching geometric construction has always been a challenge. However, Fujita et al. (2010) highlight that it is still considered as suitable vehicle for secondary school students to gain experience in learning deductive geometry. Indeed, challenging construction tasks can encourage students' mathematical argument, reasoning and proof. In addition, Cheung et al. (2010) suggest that geometric construction can be acted as application of deductive geometry. For example, through the construction of parallel lines and angle bisector, students can realize the usage of congruent triangles in real life. Therefore, it is worth for teachers to reflect the role of geometric construction in mathematics classrooms.

(II) Dynamic geometry environment (DGE)

Nowadays geometric construction can be done in dynamic geometry environment (DGE), which is a computer micro-world with Euclidean geometry as the embedded infrastructure. In this computational environment, a person can evoke geometrical figures and interact with them (Hoyles, 1993). There are many dynamic geometry software programmes such as GeoGebra, Carbi and Sketchpad. Some common anatomical features of them are navigation, interaction, annotation, construction, simulation and manipulation (Hegedus, 2005). Besides the above general features, Leung, Chan & Lopez-Real (2006) emphasized that a key feature of DGE is its ability to visually represent geometrical invariants through dragging. Dragging is the continuous real-time transformation of the figure on the screen. Baccaglini-Frank & Mariotti (2011) states that the dragging in DGE makes it different from the traditional paper-and-pencil environments, since dragging allows users to transform the original image to a sequence of new images. The changes in the image on the screen will be perceived in contrast to what simultaneously remains invariant. Leung (2014) suggests that looking for invariant in variation and using invariant to cope with variation are essences of mathematical concept development, and it is possible to bridge the experimental-theoretical gap in the DGE context, results in facilitating students' deductive reasoning through students' observation, conjecture and justification in DGE.

(III) Experiential learning in mathematics

DGE is experimental in nature, which is possible to facilitate students' learning on geometry through experiential learning. In simplest form, experiential learning is based on the concept of "learning by doing" by John Dewey, and further development by many scholars such as Carl Rogers and David Kolb. In Kolb's experiential learning theory, it emphasizes the learner's perspective and states that learning is the process whereby knowledge is created through the transformation of learner's experience (Kolb, 1984). Furthermore, Kolb suggests that learning process is a cycle involving four stages: concrete learning, reflective observation, abstract conceptualization and active experimentation. Effective learning can be seen when the learner progresses through the cycle. However, it may be difficult to introduce experiential learning in mathematics, especially in deductive geometry. De Villiers & Heideman (2014) state that when students are engaged in proving activities, they are usually guided by various sub-steps or sub-problems instead, and being pushed forward to an eventual proof of the given problem. Students have limited opportunity to explore and conduct their own conjectures in proving activities. In order to let mathematics compatible with experiential learning, a paradigmatic change and the transition from planning a content-focused course to planning an experimental learning course is required (Davidovitch et al., 2014). Furthermore, the power of play through electronic games can also facilitate experiential learning which can bridge general mathematical competence and computational contemporary culture together (Fenyvesi et al., 2015). Last but not least, experiential learning can be divided into two major categories: field-based and classroom-based. Classroom-based experiential learning can be role-playing, games, cases studies, simulations, presentations, etc. (Lewis & Williams, 1994), which are suitable to be conducted in mathematics classrooms as well.

Introduction to Euclidea

Euclidea is a free interactive geometric construction game available in multi-platform (Windows, iOS and Android), which guides the users through the basic tasks (e.g. line and angle bisectors, perpendiculars) to complicated tasks (e.g. inner/outer tangents of circles, regular hexagons and golden section) in geometric construction. The followings are brief layouts of Euclidea:

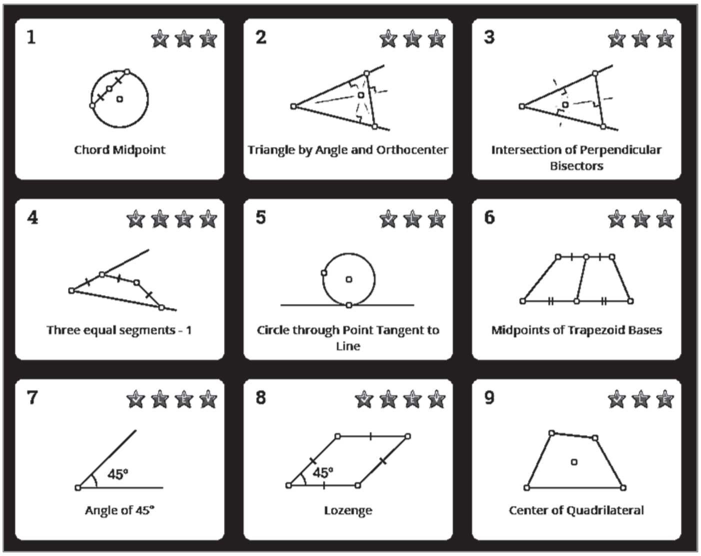


Figure 1: Examples on geometric construction tasks in Euclidea

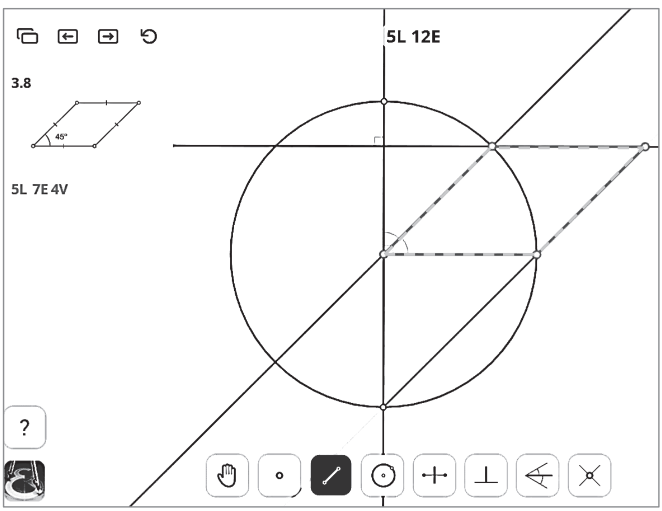


Figure 2: Constructing a rhombus with an interior angle of 45°

Teaching and learning geometric constructions has always been a challenge, especially in traditional paper-and-pencil environment. Euclidea has some special features which can reduce the difficulty of geometric construction in practice:

(I) Automatic verification of solution:

Once users construct new objects (points, segments, circles, etc.) in Euclidea, it will automatically verify whether the construction is legitimately completed. Teachers are not required to examine every step in students' construction in order to justify the deductive correctness of the construction. Hence it reduces the workload of the teachers in catering individual learner differences in the classrooms, and students can acquire instant feedback from Euclidea also.

(II) Explore mode and hints:

Once users encounter difficulties in completing the construction, they can request Euclidea to show some hints. For example, the target object needed to be constructed will be on the screen when users switch Euclidea from normal mode to explorer mode:

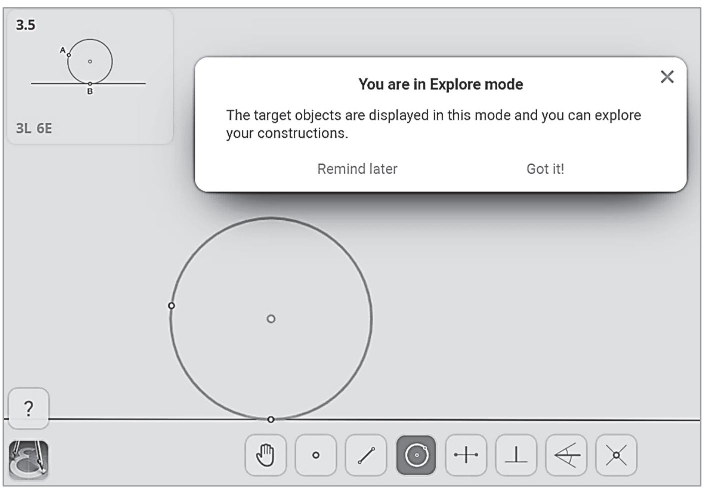


Figure 3: Target object shown in explorer mode to provide insights for students

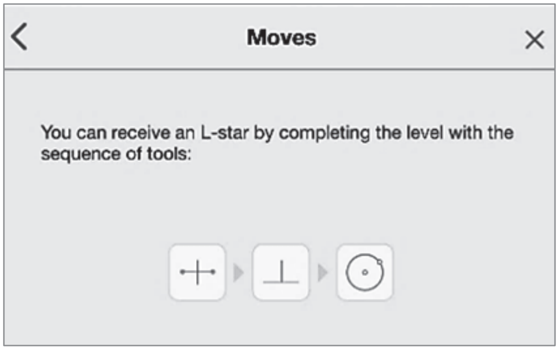


Figure 4: Procedure of construction will be shown as extra hints as students' request

Using the task of "constructing a circle through given point and tangential to given lines" as example, students can use explorer mode to examine the relationship between the given elements (two points and a line) and the target object (the circle) which may provide students with some insights for them to conjecture how to complete the construction. Furthermore, students can ask for extra hints which shows the procedure (but not in detail) on how to complete the construction. It can be acted as a scaffold for students engaging in geometric construction, and it helps teachers to cater the learners difference in the classroom.

<u>(III) Dynamic geometry in action:</u>

Euclidea allows students to drag different points to dynamically reshape the construction on the screen. It allows students to justify whether their constructions are legitimate or just visually coincides with the correct one. Using the task of "constructing a line through given point for cutting the rectangle into two equal halves" as example, if students do their construction which is not legitimate, they can realize it by reshaping the figure through dragging:

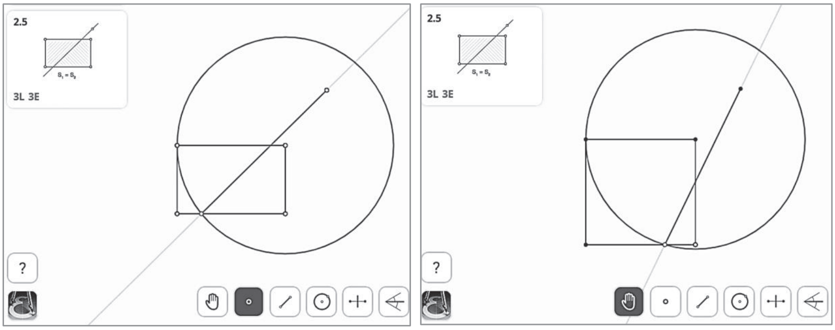


Figure 5(a)(b): Students can drag to reshape the construction (with invariant properties and relationship of geometric figures remains unchanged through dragging) to realize that their construction is not legitimate.

If a construction is legitimate, students will find that the line constructed passing through the center of the rectangle will always cut the rectangle into two equal halves during reshaping the figure. Invariant properties and relationship of geometric figures will remain unchanged throughout dragging in DGE.

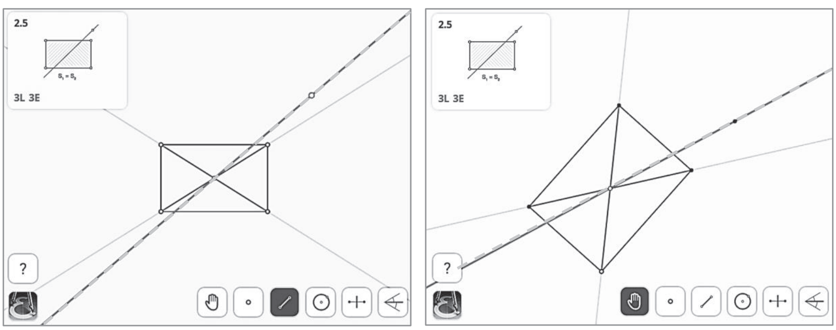


Figure 6(a)(b): Students can drag to reshape the construction (with invariant properties and relationship of geometric figures remains unchanged through dragging) to realize that their construction is legitimate.

In short, Euclidea has a great potential to facilitate experiential learning on geometry in classroom practically. Through the implementation of interactive geometric construction activities in the classrooms, students have chances to realize the application of deductive geometry. Meanwhile teachers can use Euclidea as an interactive I.T. activities which can facilitate assessment as learning.

How Euclidea facilitate experiential learning on geometry: A case study

In order to study on the play of Euclidea by students, the author setup a booth at a local secondary school. In the booth, many extend readings related to Euclid geometry were exhibited. Meanwhile a whiteboard with interactive projector had been placed which allowed students to play with Euclidea. Students were invited to participate in various interactive geometric construction, and they were encouraged to discuss and work collaboratively to complete the construction throughout the play.

Figure 7: A booth with Euclidea setup in a secondary school

Figure 8: Collaborative working of students with geometric construction

Here is a scenario on how two secondary five students discussed and worked collaboratively to construct a circle through given point and tangential to given lines:

Student A: *Firstly we need to locate the center of the circle, or else we cannot construct the circle.*

Student B: (Observe the problem for a while) ... *I think this problem can be solved by using some theories about circles.*

(Think for a while)... *Yes, the perpendicular bisector of the segment joining the two given points will probably pass through the center.*

Student A: *But how we locate the center of the circle on the perpendicular bisector?*

Student B: *I have no idea ... but let me construct the perpendicular first.*

Student A: (After constructing perpendicular bisector)

In order to locate the center, we need to construct one more line that intersect with the perpendicular bisector at the center of the circle... I have no idea, let's switch to explorer mode to get some hints.

Student B: (After switching to expolrer mode)

How about a line perpendicular to the tangent? It seems that the center of the circle lies vertically above the given point on the targent.

Student A: *Yes, let's constuct the line ...* (constructing the perpendicular) … *Yes, it passes through the center also. Now let's switch off the hints and make the construction again ... and construct the circle also…* (constructing the circle) … ("Task completed" message from Euclidea) *it's done!*

In the discourse of the two students, different stages (e.g. observation, justification, etc.) during construction can be easily be highlighted. The observation (watching), justification (thinking) and construction (doing) that the students proceeded can be modelled by Kolb's experiential learning cycle:

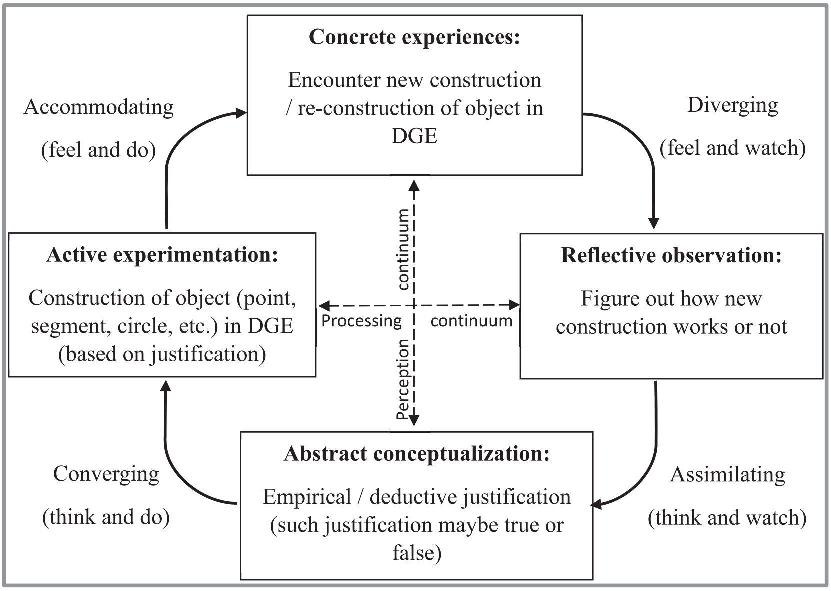


Figure 9: Experiential learning cycle of students in interactive geometric construction

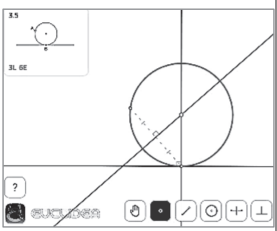
Traditional way of teaching and learning Euclid geometry stays in the perception continuum: students experience a new geometric problem (concrete experience), then they are guided by the authority (teachers and textbooks) to work out a solution and justify it (abstract conceptualization). Students are encouraged to reproduce formal mathematics arguments (proof) and almost never engage in the reflection on their own arguments. Euclidea can provide students an opportunity to learn geometry in processing continuum (reflective observation and active experimentation) in addition to perception continuum. The following table shows how the working of the two students in the geometric construction related to both continuum:

Table 1 Students' working and corresponding stage in experiential learning theory

	Students' working	Stage in learning cycle	Continuum
1.	Try to locate the center of the circle at first glance after observing the problem	Reflective observation	Processing
2.	Realize that perpendicular bisector must pass through the center of the circle (by circle theorems they learnt)	Abstract conceptualization (deductive justification)	Perception
3.	Construct the perpendicular bisector	Active experimentation	Processing
4.	Encounter perpendicular bisector as a new object on the screen	Concrete experience	Perception
5.	Try to construct one more line which will also pass through the center	Reflective observation	Processing
6.	(Both students had no ideas to proceed to the next step)	-	-
7.	Switch Euclidea to explorer mode in order to get more hints	Active experimentation	Processing
8.	Encounter the target circle (with its center) as new objects on the screen	Concrete experience	Perception
9.	Observe that the center lies vertically above the given point on the tangent	Reflective observation	Processing
10.	Realize that the perpendicular from the tangent may pass through the center	Abstract conceptualization (empirical justification)	Perception
11.	Construct the perpendicular	Active experimentation	Processing
12.	Encounter the perpendicular as a new object on the screen	Concrete experience	Perception
13.	Verify that the constructed lines intersect at the center	Abstract conceptualization (empirical justification)	Perception
14.	Switch off the hints and complete the rest of the construction	Active experimentation	Processing

The above interplay between processing and perception continuum in the experiential learning cycle can be treated as enactive proving activity in geometry. Enactive proof is considered as the most primitive level in cognitive development of representation of deductive reasoning, and it involves carrying out a physical action to demonstrate the truth of something (Tall, 1998). However, justification in enactive proof is not always deductive. Marrades & Gutierrez (2000) suggest that there are two types of justification in such proving activities: empirical and deductive justifications. Empirical justifications are based on the use of examples (randomly chosen or selected purposefully), while deductive justifications are based on abstract formulations of properties and of relationships among properties. From Table 1, it shows that the two students involved both types of justification during their construction:

Table 2　Examples of deductive and empirical justification in Euclidea

Task	Deductive justification	Empirical justification
Constructing circle through given point and tangential to given lines 	Students recalled the geometric theorems that perpendicular bisector of a chord of the circle will pass through its center, then constructed the perpendicular bisector (working #2 – 4).	Students observed (via explorer mode) that the center of the circle lies vertically above the given point on the tangent, and then constructed the perpendicular (working #7 – 13).

By the observation in explorer mode in Euclidea, two studnets noticed that the line joining the center of the circle and the point of contact between the circle and the tangent is perpendicular to the tangent (actually it can be dervied from the thoerem “tangent ⊥ radius” but somehow both students forgot this theorem that they learnt in last year). Then they did the construction and Euclidea prompted that their conjecture was legitmate. It was an example of enactive proof with empirical justification. Enactive proving activity starts from interaction of learners with environment, which is coherent to the main idea of experiential learning. Tall (1999) highlights that through the perception and action on objects (active experimentation), leaners can acquire concrete experience which probably facilitates them to develop new mathematical ideas.

Conclusion

This article shows the possibility of using Euclidea, an interactive geometric construction game in dynamic geometry environment (DGE), as facilitator of experiential learning on geometry. Throughout the geometric construction, students will engage in active experimentation, concrete experiences, reflective observation and abstract justification as stated in Kolb's experiential learning theory. Such process induces the interplay of processing and perception continuum during geometric construction and can be regarded as enactive proving activity which will facilitate empirical justification of students on Euclid geometry. However, more dragging strategy (especially dragging test) should be encouraged in geometric construction in DGE in order to initiate reflective observation of the students during their construction, in order to transit the justification of the students from geometric visualization (empirical in nature) to formal axoimatic Euclidean geometry (deductive in nature). Teaching and learning geometric construction is always challenging, but it is believed that Euclidea have a great potential to minimize those challenges and make geometric construction be practical to be held in mathematics classrooms.

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Author's e-mail: patrick_chung@live.com