The Effect On Students' Mathematical Beliefs By Integrating History In The Mathematics Classroom

CHEUNG Wai Shan
True Light Middle School of Hong Kong
WONG Ngai Ying
The Education University of Hong Kong

Analyses of 47 empirical studies on the use of history in mathematics teaching revealed that only a few involve the evolution and development of mathematics. Even fewer concern students' conceptions of mathematics. This is precisely the aim of the present study to fill this gap, taking function as the context. Teaching in a treatment group, versus a reference group, of 72 Secondary 4 students showed that the introduction of history did make a difference in student's various mathematical abilities as well as their beliefs in mathematics.

Introduction

The use of mathematical history in teaching has long been advocated (Fauvel, 1991). At the start, more attention was paid on theoretical premises. Later, teaching capsules were designed and eventually, empirical studies have been carried out.

Jankvist (2009) categorized empirical studies by their uses, viz. including using history as an affective and motivational tool (Type I), using history as a cognitive tool (Type II) and historical parallelism (Type III). Forty-seven empirical studies over the past 30 years (1977-2013) were identified by GoogleScholar®, it was found that most of them fell into either Type I (30 studies) or Type II (10 studies). Only a few (7 studies) touched upon the third type.

However, in the lens of recapitulation (see below), investigations under historical parallelism are unavoidable. To look into the Type III use of history, it is not enough to tap students' changes in their cognitive or affective learning outcomes. Exploring how students change in their views or beliefs in mathematics is indispensable. This is precisely the theme of the present study.

The use of history and the beliefs in mathematics

The use of history in mathematics teaching has been widely discussed. In Fauvel's (1991) frequently cited piece, various reasons for the use of history were offered. Increasing the motivation to learn, giving mathematics a human face, enhancing the topic presentation in the curriculum, and showing students how the development of concepts helps their understanding are some of them. Yet, there is another unique aspect in the use of history (with differs from the use of other means like the use of games, activities or ICT), which concerns the notion of recapitulation.

The idea was first put forward by a German biologist Haeckel in 1874. In brief, "the zoologists maintain that in a brief period the development of the embryo of an animal recapitulates the history of its ancestors of all geological epochs" (Furinghetti & Radford, 2002). This was further elaborated by M. Klein in the 1930s. In parallel with "Ontogeny recapitulates phylogeny", students develop a mathematical concept better if it is unfolded according to how the concept was evolved in course of historical development (Siu & Siu, 1979).

As said, the role of mathematics beliefs in mathematics education has to be considered (Leder, Pehkonen, & Törner, 2003). Alongside with the increasing number of literatures on mathematical beliefs internationally, a series of studies were conducted in a Hong Kong research team. It was found that students (those in Hong Kong and in the Chinese mainland) saw mathematics involves manipulations and a subject of *calculables*, though they also saw that mathematics is useful, and involves thinking. They also think that the task of mathematics problem solving is to seek appropriate routines so that they can execute step by step. A reliable instrument, the *Conception of Mathematics Scale* was thus developed, which yielded fruitful results (Wong, Ding & Zhang, 2016; Zhang & Wong, 2015).

Research focus

Historical parallelism (Type III) was the focus of this study. The notion of function was the context of the (mini-) curriculum and how it evolved historically, from Babylonian Table to discontinuous function, were considered. The change of students' affects and beliefs were explored after the teaching, in which historical parallelism was incorporated. The precise research questions were to investigate the changes in students' mathematical cognitive, affective outcomes as well as beliefs after the implementation of the curriculum. There were two reasons for choosing *function* as the context. It covered a wide range of topics like equations, curves and trigonometry. In addition, it had a long history, in which its notion underwent a number of changes (Siu, 1995; Zhang & Wong, 2015).

Methodology

Curriculum design

A curriculum including lesson plans and worksheets was carefully designed, which then was undergone the process of expert validation. A mathematics educator who holds a doctoral degree in mathematics education, and two senior mathematics teachers were invited to comment on the curriculum. The flow of the revised curriculum is depicted in Figure 1.

Research setting and participants

The curriculum lasting for 2 weeks was carried out in normal classroom settings in 2013. Two classes, each of Secondary 4 students in the same girls' school were recruited as the experimental (N=38) and the reference (N=34) groups. To guarantee similar baselines for comparison, the mathematics pre-test scores (cognitive, affective and beliefs: Table 1) of these two groups were compared by the use of t-test and no significant differences were found. And to avoid the halo effect, teaching of both groups was carried out by the same teacher other than the researcher. Mixed methods (quantitative and qualitative methods) were used. Pre-tests and post-tests of various measurements were administered in both groups. To collect thicker data, in depth interviews were conducted among six students (2 with high pre-cognitive-test scores, 2 medium and 2 low – both in the treatment and control groups: a total of 12) each from the treatment and reference groups.

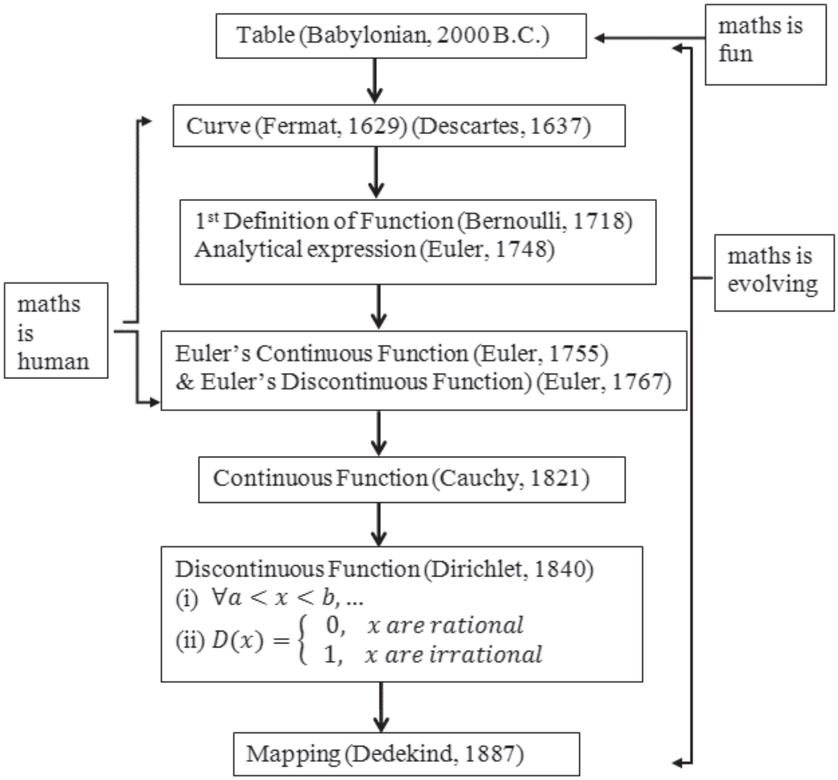

Figure 1: Curriculum Flow

<u>Instrument</u>

Cognitive test: A specially designed test was used in the pre-test to measure the prerequisite knowledge while the post-test contained items corresponding to what was taught in the curriculum. The pre-test and post-test each comprising 7 questions. Each question carries 2 marks, making the full mark 14.

Aiken's Mathematics Attitude scale (Aiken, 1979), which was used several times and yielded promising reliability indices, was employed to measure students' affects. It comprises the subscales of 'enjoyment', 'motivation', 'importance' and 'freedom from fear', each containing 6 items.

Conception of Mathematics Scale was used for beliefs. It comprised the subscales of 'mathematics is calculation' (14 items), 'mathematics involves thinking' (6 items) and 'mathematics is useful' (6 items). Since the use of history heavily involves the notion of mathematics is felt being human and that it evolved in history. After consulting literature and various experts, no relevant questionnaires were available to date. There was a need to develop one for our use. Keywords from Ernest (1989), Philipp (2007) and Schoenfeld (1992) etc. were extracted. The items were constructed accordingly and tested among 220 students. After revision, we came up with a scale that consisted of 'mathematics is human' and 'mathematics is evolving', with 13 and 10 items respectively. The reliability index Cronbach alphas were 0.65 and 0.77 respectively.

For the interview, questions focused on students' views on the nature of mathematics. Sample questions were, "Is graphical solution mathematics?" and "Can astrology and divination considered as mathematics". For the treatment group, the responses on the introduction of history in the mathematics lessons were also solicited.

Result

Quantitative data

A t-test on the post-tests between the treatment and the reference groups were performed and the results are shown in Table 1. Results revealed that though there were no significant changes in the cognitive test, there were, however, significant changes in 7 out of the 10 affective subscales. Students in the treatment group, when compared with the reference group, found, after the experiment, mathematics was less as a subject of *calculables*, but a more useful and enjoyable one. They were more motivated to mathematics, saw mathematics' importance, and had more freedom from fear. They also saw mathematics as an evolving discipline. This is encouraging.

Table 1 t-tests on pre-tests and post-tests

Mean	Pre-test			Post-test		
	Treatment group	Reference group	t-value	Treatment group	Reference group	t-value
Cognitive test	11.54	11.21	0.94	10.90	10.00	1.60
Calculation	3.40	3.46	-0.40	3.15	3.42	-2.43*
Thinking	4.26	4.25	0.11	3.96	4.09	-1.14
Useful	3.45	3.64	-1.12	3.71	3.18	3.60**
Enjoyment	3.53	3.75	-1.47	3.71	3.09	4.58***
Motivation	3.46	3.58	-0.76	3.60	3.04	3.68***
Importance	3.64	3.66	-0.16	3.79	3.26	4.58***
Freedom from fear	3.16	3.36	-1.08	3.33	2.90	2.53*
Math is human	3.18	3.22	-0.37	3.16	3.15	0.06
Math is evolving	2.88	2.93	-0.44	3.33	2.99	2.85**

*$p < 0.05$ ** $p < 0.01$ *** $p < 0.001$

Interview

Most of the students from the treatment group gave positive responses, they welcomed the use of history in the mathematics class. A student said, "I used to apply formulas without understanding their origins, these lessons let me know more how the formulas were evolved to their present forms". All 6 of them viewed that mathematics was not a set of truths but an evolving one. One said, "It is possible that someone may use another way to express quadratic equations, or someone may even overturn the whole thing". By contrast, only 2 students (both with high mathematics scores) in the reference group thought that quadratic equation may not be regarded as mathematics after ages and the rest of the four students thought otherwise.

Most (10) of the students, in either the treatment or reference groups, expressed that formulas were not essential in mathematics proofs. A student said, "Other methods (such as graphs/figures) can also be used for mathematics proof, which is not confined to representing by an abstract formula". The rest (2 from the treatment group with high mathematics scores) emphasized that mathematics proofs should contain formulas. In her own words, "Mathematics proofs cannot be valid without formulas”. In a sense, it is another form of ‘mathematics seen as a subject of *calculables*’. Mathematics should be manipulatable with a formula.

In fact, 9 students thought that mathematics was a subject of *calculables.* A student mentioned, “One has to calculate. Mathematics should involve calculation. Mathematics without calculation is no more than copying answers”.

Students invited to the interview were also requested to draw a concept map circling around the notion ‘mathematics’. In general, concept maps from the treatment group showed more linkages between components. Typical concept maps from the treatment group and the reference group are depicted in Figure 2. It is worth noticing that the concept map drawn by the treatment group showed a broader understanding of the nature of mathematics and reflected that mathematics was not only a set of truths.

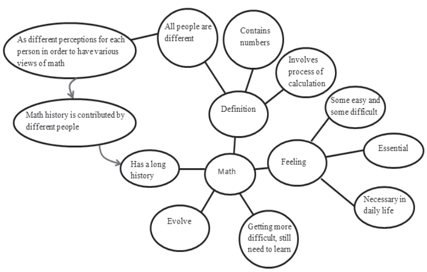

Figure 2: Typical concept maps of students from the treatment (left) and reference (right) groups

<u>Stories of six students</u>

In the interview data, we noticed that though by and large, those in the treatment groups differed from those in the reference group, the mathematics backgrounds, via their mathematics scores, might count. To obtain a deeper understanding of the picture, we portrayed the ‘stories’ of 6 students (3 each from the treatment and reference groups). Here were the brief outlines.

Ann (Treatment Group with a high mathematics score: 14/14): From the questionnaire, Ann was generally positive towards mathematics. She reflected that she gained a deeper understanding of how the notion of function was evolved. However, since she knew that already by self-reading, she found the pace in class a little bit too slow.

Betty (Reference Group with a high mathematics score: 13/14): Betty was also positive towards mathematics. Though she was situated in the reference group (Note that the teacher in this group did not touch upon history of mathematics in class), she knew that mathematics was a static set of truths through her self-study.

Carol (Treatment Group with a medium mathematics score: 12/14): Carol did not have an ill feeling towards mathematics, but neither was she very fond of it. She reflected that the inclusion of history in the mathematics class initiated her interest to visit the library. Her view towards mathematics was also broadened.

Dora (Reference Group with a medium mathematics score: 11/14): From the questionnaire, Dora changed her feelings towards mathematics to the negative side. In the interview, she reflected that mere introduction of definitions and formulas made her find mathematics not as interesting as before.

Emily (Treatment Group with a low mathematics score: 9/14): Emily was quite negative towards mathematics. In the interview, she reflected that due to her relatively low mathematics standard, if the mathematics class was not interesting enough, she became more passive. She disliked boring lessons. However, she emphasized that the use of history did add fun to the mathematics class.

Fanny (Reference Group with a low mathematics score: 9/14): Fanny was pretty negative towards mathematics. She reflected in the interview that mathematics was tedious. She did not have much knowledge about the nature of mathematics.

From these stories, it was clear that the use of history and the students' mathematics background interact with each other to make an effect. Though apparently the introduction of history made a difference, the effect would be more salient among the more able students. Not only that, able students gained a lot more, whether with or without the use of history, with their out-of-class self-learning. This is in fact not a surprise.

Discussion

Summary of results

The present study adds to the literature of empirical investigations on the use of history in mathematics, which only a few have been done. It not only echoes what was done previously, for instance by Lit, Siu, & Wong (2001) but has gone further. The introduction of history to the mathematics class does not hamper students' mathematics (cognitive) performance but enhances their interests in mathematics. Not only that their views and beliefs in mathematics are changed. This is of utmost importance to their learning. As said as in Zhang & Wong (2015), the richer the students' experience, the broader their conceptions towards mathematics, and in the long run, more flexibility in solving mathematics problems.

Directions for further research

It is now common knowledge that the use of history enhances mathematics learning. But we believe that learning outcomes have a much broader sense than conventional test scores. Besides whether students love mathematics, how they see mathematics is yet another important aspect that we need to attend. In the present study, with the consideration of historical parallelism, the change of students' beliefs is prominent. We need to move a step further to investigate how it works with other target groups (e.g. boys, elementary students) and topics (e.g. more computational topics). The present study also opens the possibility of the interaction between the use of history and student's background, their mathematics standards in particular. It is not just a matter of how much history works with the more and less able students but how we make it work for these different groups, making mathematics learning more fun as well as more effective.

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First author's e-mail: tlcws@tlmshk.edu.hk