Improving Mathematics Teaching by Using Manipulatives

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Teachers are always interested in looking for ways to improve their teaching and to help students understand mathematics. Research in England, Japan, China, and the United States supports the idea that mathematics instruction and student mathematics understanding will be more effective if manipulative materials are used (Canny, 1984; Clements & Battista, 1990; Dienes, 1960; Driscoll, 1981; Fennema, 1972,1973; Skemp, 1987; Sugiyama, 1987; Suydam, 1984). Mathematics manipulative is defined as any material or object from the real world that children move around to show a mathematics concept.

Manipulative materials in teaching mathematics to students hold the promise that manipulatives will help students understand mathematics. At the same time as with any "cure", manipulatives hold potential for harm if they are used poorly. Manipulatives that are improperly used will convince students that two mathematical worlds exists - manipulative and symbolic. All mathematics comes from the real world. Then the real situation must be translated into the symbolism of mathematics for calculating. For example, putting three goats with five goats to get eight goats is ~~the real~~ world situation but on the mathematics level we say 3+5 = 8 (~~Read three~~ add five equals eight). These are not two different worlds but they are in the same world expressing the concepts in different ways.

What are manipulative materials? Manipulative materials are concrete models that involve mathematics concepts, appealing to several senses, that can be touched and moved around by the students (not demonstrations of materials by the teacher). The manipulative materials should relate to the students' real world. For example, the use of an abacus is not something that is used in Malawian daily life. Instead stones, eating utensils, tins, beans, apples, peanuts, sticks, etc. would be more appropriate.

Each student needs material to manipulate independently. Demonstrations by the teacher or by one student are not sufficient. With students actively involved in manipulating materials, interest in mathematics will be aroused. Manipulative materials must be selected that are appropriate for the concept being developed and appropriate for the developmental level of the students. For example, one stick may be

placed on a place value chart in the ones place. However one stick should not be placed in the tens place. Instead a package of ten sticks bundled together with string or an elastic should be placed in the tens place. Students need to realize and conceptualize the idea of tenness. The same is true for the concept of the hundreds place, a bundle of 100 identical things should be used. As the students' concept of place value develops, then single sticks can be used for place value of numbers with greater value.

Good mathematics manipulative materials are durable, simplistic (easily manipulated), attractive (to create interest), and manageable. A systematic method should be developed for storage and distribution of materials. Baskets or boxes are convenient for storage and distribution purposes.

Using manipulative materials in teaching mathematics will help students learn:

1. to relate real world situations to mathematics symbolism.
2. to work together cooperatively in solving problems.
3. to discuss mathematical ideas and concepts.
4. to verbalize their mathematics thinking.
5. to make presentations in front of a large group.
6. that there are many different ways to solve problems.
7. that mathematics problems can be symbolized in many different ways.
8. that they can solve mathematics problems without just following teachers' directions.

If mathematics is taught using maipulative materials, then the methods of evaluating mathematical achievement must also change. Just calculating correct solutions to mathematics problems is not sufficient. Concept development and understandings should be valued more highly. Effective use of mathematics manipulatives contributes to conceptualization and understanding. Evaluation of students' mathematics is changing from tests and testing to assessment. Assessment is much broader than testing or evaluation. For teachers to assess students' understanding of concepts, different techniques of evaluation will be needed. Teachers will receive more insight into students' mathematics understanding by:

1. listening to students' talk about their mathematics thinking.
2. observing students working individually and in cooperative

groups.

3. asking why and how questions rather than asking:
   a. yes or no questions.
   b. for results of calculating activities.
   c. for answers.
4. having students write a solution to a problem rather than by only responding with correct or incorrect values.

Paper-and-pencil method of assessment limits the scope of student evaluation. Requiring students to defend their mathematical reasoning provides insight into the development of the students' thinking skills. Observation of students' functioning within a group will provide data for assessment. The teacher will move around the classroom observing how students are working and interacting.

To facilitate collecting assessment data, different type of questions will need to be asked by the teacher. The traditional questions which focus on calculating and correct answers will change to:

1. how and why questions.
2. probing questions to stimulate the thinking process of the students.
3. having students write responses to mathematics problems.
   a. Integrates writing with mathematics
   b. Numerical values are not sufficient for answers to mathematics problems.
   c. Presents an opportunity for reflection, which is conducive to students' cognitive development.
   d. Helps identify students having mathematical difficulties.
   e. Helps identify the conceptual level of development of the students.

Some examples of appropriate questions and responses for students might be:

1. How do you know that ?
2. What would happen if ... ?
3. Why do you suppose ... ?
4. What makes you think your answer is correct ?
5. How could you prove your answer is correct ?
6. Could you express your answer in a different way ?
7. What is another way to solve the problem ?
8. How many different ways can you find to solve problems ?

9. How can you convince the other members of your group that your way is the best method to solve the problem ?

Conclusions

Mathematics teachers are learning to direct their attention toward the facilitation of students' understanding and conceptualization rather than drill and practice of rote procedures. The use of manipulative materials in mathematics classrooms supports this attempt. Incorporating the use of manipulative materials with an emphasis upon the thought process of students provides an opportunity for the teacher to assess and meet the needs of primary school students as they construct personal mathematical knowledge.

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