Estimation of Large Quantities in a Primary Five Classroom

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Introduction

Estimation is the process of guessing the approximate value of a number. With reference to the Hong Kong Primary Mathematics Curriculum Guide (2000), estimating the number of a large quantity of objects is one of the suggested learning objectives of Large Numbers (5N1) in the dimension of Number at Key Stage 2 (CDC, 2000, p.38). To have an overview of the mathematics curriculum, estimating the quantity of objects has been introduced after counting in groups of two, five and ten in the learning unit of Numbers to 100 (1N4) and counting in groups of fifty or hundred in that of 3-digit numbers (2N1) at Key Stage 1 (CDC, 2000, p.23 & 25).

On the other hand, estimating the answers, providing quick estimates of the result of a calculation, usually by rounding-off the numbers when performing numerical computations, is another learning objective that requires in the mathematics curriculum. It is about simplifying calculations. In the learning dimension of Measures, estimation is also emphasized. Students should have been encouraged to estimate certain quantities such as lengths, weights, heights or capacities, duration of time for different activities, perimeters, areas and volumes in specific examples before actual measurements. Furthermore, estimating the average from block graphs and bar charts in the learning dimension of Data Handling is another suggested learning objective that requires different estimation skills to be achieved in the primary years. In short, estimation of quantities, numerical estimation, estimation in measurement and estimation of average from graphs are the learning objectives that embedded across different learning dimensions in the curriculum design. Owing to its complexity, various skills and knowledge of estimation would be introduced to the students through the mathematics curriculum. In the long run, students

would develop the reasoning ability to handle different estimation problems in daily life.

To focus on how students estimate the number of a large quantity of objects at Key Stage 2, the following passage is a description of an estimation activity held in a Primary Five classroom. How did the students understand about estimation, especially the estimation of the number of a large quantity of objects? How did they get a close estimate? How did they justify the strengths and weaknesses of their strategies? Could the errors be reduced? All these would contribute to further considerations about the learning and teaching of estimation.

Background of the activity

For the learning dimension of Number in the mathematics curriculum, students have obtained the skills of estimating the quantity of objects by counting the objects in groups of two, five and ten in Primary 1. To further develop the skills, they have been trained to count in groups of fifty or hundred for estimating the quantity of objects in Primary 2. However, is estimating large quantities exactly aligned with what students learnt at Key Stage 1, with only a difference of the quantity itself? Are there other alternative strategies to estimate the quantity besides counting in groups? What does estimation really mean? If we are not trying to get the exact number, how can we evaluate the estimate by actual counting?

According to the exercises designed in some local textbooks, students estimate large quantities of objects shown by pictures, for example, large number of books on a shelf, people at a concert hall or number of words in a selected passage, etc. The suggested method of estimating such quantities is to divide the picture into several regions, count the number of any one of the regions, and multiply that quantity by the total number of regions. In general, students have no difficulty to follow these standard procedures to work out the estimate. However, the ability to estimate is an important skill in daily life. It is plain that such large quantity is seldom presented on a plane surface in real life. In other words, students should be exposed to more authentic situations, so that

they could suggest ways of estimation, estimate the quantity of objects with reasonable judgement on the facts, and evaluate their approaches of estimation, rather than merely following some steps to come up with an answer.

The following classroom activity aimed at providing students with hands-on experience to estimate the large number of beans. Going over the process of estimation, students were encouraged to make assumptions and examine the reasonableness of the results. It was hoped that students would explore their own methods to solve the problem. They were not only to work out the answer, but also to justify their strategies and the limitations to get a close estimate.

How students estimated the number of beans

A group activity of estimating a large number of beans was carried out in a Primary Five classroom. The following description reported how the students found the estimates of the number of beans.

1. Preparation of the activity

Teacher prepared six bags of beans and poster-sized paper as backing sheets for the estimation activity. The actual number of beans was also found for reference in the discussion. Time limit was not fixed as long as the students could get their estimates.

2. Warm-up discussion

Teacher showed the students a bag of beans at the beginning of the lesson. They were asked to estimate the number of beans in the bag by direct observation. Some students estimated with wild guesses. Their estimates were either much bigger or smaller than the actual number, and their peers quickly rejected some of these unreasonable estimates. Some students felt hard to make a guess. In order to get a better estimate, they started the group discussion.

3. Group activity – Estimating the number of beans in the bag

The group activity was carried out in a double-lesson of 70 minutes of a Primary Five classroom. The class was divided into groups of five. A bag of beans and a piece of poster-sized paper as backing sheet were given to each

group. Students were free to use any methods for estimating the number of beans in the bag.

3.1 By actual counting

It was assumed that students would get an estimate or an approximate number with a guess, but many students were much eager to find the actual number rather than an estimate. Two out of six groups used counting method. Students counted the number of beans in groups of ten (see Figures 1 and 2). This seemed to suggest that students had applied their previous knowledge to estimate a quantity by counting in groups. Instead of finding a number close to the exact number, they had only focused on how to get the exact number by actual counting.

Figure 1

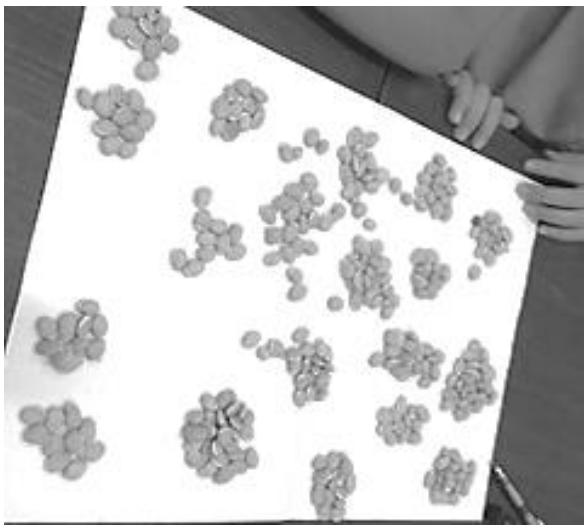

Figure 2

There was an unexpected outcome. Although the poster was supposed to serve as a backing sheet, one group of students was attracted by the coloured-grid printed on one of the posters. The teacher did not notice the pattern on the poster beforehand. The students placed the beans one by one on the grid. Eventually, they estimated the number by counting the rows of beans (see Figure 3). During the activity, the teacher had advised the students to try again with another way, but they replied that this was *the method*. Frankly, the grid might have misled the students to estimate the number by actual counting. If there was no grid, they might think of other strategy. Instead of putting the

beans into groups, the “grid” provided a tool that made counting easier because it helped to put the beans in good order. It was even more accurate to estimate with the help of the grid. This was also an example of actual counting.

Figure 3

3.2 By dividing the beans into parts and counting the number of one part as a reference

The other three groups had similar approaches to estimate the result. Firstly, they spread the beans on the paper. Next they divided the beans into eight or ten parts. To find the estimate, students counted the number of one part, and multiplied that quantity by the number of all parts (see Figures 4 and 5). Basically, this method was similar to the suggestion in most textbooks. However, in the hands-on experience in working out an estimate in this real-life situation, there were some major differences:

(i) the students had to start the estimation process by first placing the beans on a plane surface themselves;
(ii) making judgement on dividing the beans into certain parts; and finally
(iii) obtaining the estimate by using the number of one part as a reference.

Figure 4

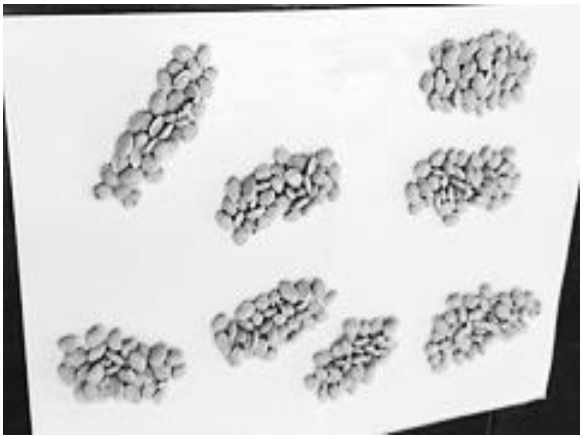

Figure 5

4. Results of the estimates

The following table shows the students' estimates by the two approaches.

1. Actual counting	228 227 231
2. Using the number of one part as reference	295 (divided into 5 parts) *224 (divided into 8 parts) 270 (divided into 10 parts)

* The exact number of beans was 230. Apart from using the actual counting, one group had indeed got an estimate 224 which was very close to the actual number.

Reflection on students' performance in the activity

1. Clarification of the meaning of estimation

Getting an exact answer is always stressed in mathematics. With reference to the students' performance in the activity, students did make some wild guesses. But the counting behaviours of half of the groups suggests that students tend to feel more comfortable to count in order to get a close estimate, that is, the exact answer, even though they noticed it was time consuming to do so.

Strictly speaking, this result would be closest to the actual number for it was already an actual count. While students had learnt the method of estimating

the quantity by counting in groups early in Primary 1 and 2, it was not surprising to find that about half of the class used this strategy to estimate the quantity. However, students should be reminded that to estimate a quantity was not actually counting. In real life situations, large numbers were usually not easy, and probably could not be counted by hand. For examples, *How many boxes of tissue paper are consumed in Hong Kong per year? How many people go to the Flower Market before the Lunar New Year comes?* Actually, making mistakes or being unable to find the exact answer is expected and acceptable as there are inevitably errors in estimation. As mentioned by Weinstein & Adam (2008), the ability to make rough, common-sense, estimates starting from just a few basic facts, analyse the problem and break it down into smaller pieces is the path towards the final estimate. Thus, more emphasis should be put on elaboration and clarification of the concept of estimation for the students is needed so as to let students recognize the importance of applying the skills of estimation in daily life.

2. Suggesting ways to reduce errors in estimation

It is believed that the knowledge of handling or estimating large numbers would, in fact, cover a wide range of aspects in real life from the simple to the complex. There are many factors affecting the results to come up with a reasonably close estimate. Every estimation method involves estimation errors.

During the follow-up discussion, students reflected on how they had estimated the number of beans. They reviewed that the choice of any part of the beans was a determining factor of the estimation. Taking an example of dividing the quantity into ten parts, any 1 more bean or less in that chosen part would bring an estimate of 10 more or 10 less than the actual number. It was delighted that students could rectify themselves and became more careful in the process of estimation. They needed to check whether the beans were lain properly on the surface, and how the beans could be divided more evenly in order to reduce the error. In the discussion, some students recalled that they had not placed all the beans flat enough on the paper. Some regions were bigger than the others. It was difficult to choose which part as a reference. At the moment, a student made a good suggestion of drawing 10 boxes of the same size on the paper first before

placing the beans on it. Once the beans were arranged into the boxes, they could adjust them more evenly since the number of beans on each patch would be bounded by the size of the box.

Compared with actual counting, this approach is a more efficient strategy of estimation, especially when larger quantities are involved. Estimation by breaking down the large crowds of objects into small regions was also common in daily life. Among the groups, it was so glad to see that one group obtained a very close estimate with just a small difference of six beans. On the contrary, those who actually counted the number of beans failed to get the exact number as they had expected, but interestingly, they came up with three different results, 228, 227 and 231. This reflected that even by counting the beans one by one, students would make mistakes in terms of accuracy.

3. Implications on teaching

The knowledge of getting a close estimate was the key objective to be achieved in this activity. It was found that students hesitated to give an estimate by direct observation at the beginning of the activity. In order to allow students to guess more confidently at the start, teachers should make explicitly to the students that errors were expected and accepted in the process of estimation. Any estimates, be they too big or too small, could be good start to proceed to a better estimate. During the activity, close observation by the teacher was needed for further assistance and clarification of misconceptions. To develop students' reasoning ability, teachers should guide them to treasure the analysis and reasoning process towards the final result, encourage them to explain and justify the strategies for their estimates. This would also help in creating an atmosphere to review and reflect on their strategies to solve the estimation problem.

Concluding Remarks

Students enjoyed this activity very much as it was similar to a guessing game. Without doing any tedious computation, they found the estimates. What lied beyond a close estimate was a chain of analysis with good reasoning and careful judgement that the teacher expected students to recognize. Besides introducing the method of estimation, it was hoped that students could develop

the ability to reason and think critically on the limitations of the strategy during the follow-up discussion. All around in our daily life, examples of estimation problems with large numbers are indeed far more complicated than what we have tried in the lessons. As there is no single way to handle large numbers, let this activity become a start for young students to work more on estimating them.

References

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