Integrating History of Mathematics in Teaching and Learning Logarithm: A Case Study

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Usually, logarithm is introduced to students by its definition "If $x = 10^y$, then $y = \log x$". Although it is easy and straight forward for students to follow, most of students do not know why they need to learn logarithm and what is the importance of logarithm in mathematics. By reference to the history of development of logarithm, I had tried to implement a student-centered exploratory activity for the students in learning the concept and properties of logarithm. The lesson started from the following question:

> *"We know that if $10^x = 1$, then $x = 0$; similarly if $10^x = 10$, then $x = 1$. But if $10^x = 2$, what will be the value of x? Moreover, how about if $10^x = 3$, $10^x = 4$, etc. … Can you solve these exponential equations?*

Few students said that it is impossible to solve these equations, as it did not make sense for $10 \times 10 \times 10 \ldots$ for x times to be equal to 2 or 3! However some students recall that they have learnt $10^{\frac{1}{2}} = \sqrt{10} \approx 3.16$, hence they tried to observe the following pattern and guess that the values of x should be something between 0 and 1:

x	0	?	?	0.5	?	…	1
10^x	1	2	3	3.16	4	…	10

At that moment the story of the mathematician John Napier and the idea of logarithm were introduced to them:

> *"It is difficult to find the values of x in the table above, however a mathematician called John Napier, who was born 500 years ago, had found the values of x above, without the help of computer or calculator! Finally he made it and recorded his finding into a table called Logarithm Table"*

Students were curious at that moment. Afterwards a worksheet was given to them (refer to Appendix I). Since it seemed to be too difficult for students to understand the way to find the values of log 2 and log 3 at the beginning, therefore their values were given to students and I told them the values of log 2 and log 3 would be found later. Then the students were guided to find the values of log 4, log 5, etc… , as they could still remember the law of indices they had learnt in last year, they could follow the guidelines in the worksheet successfully and use the similar way to find the value of log 6 (refer to Figure 1).

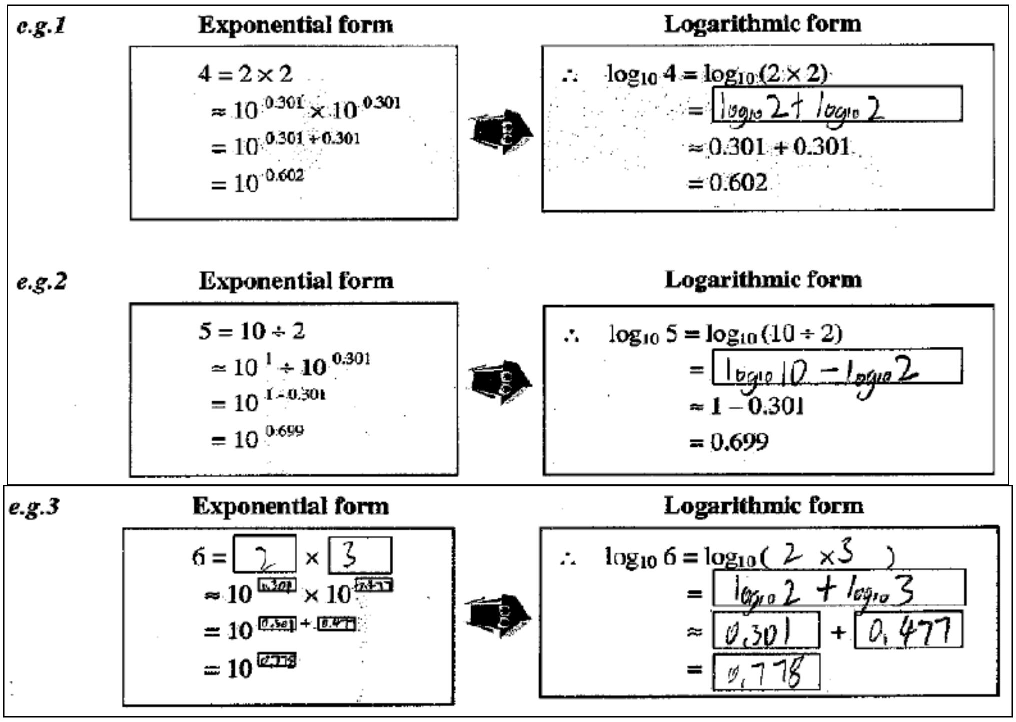
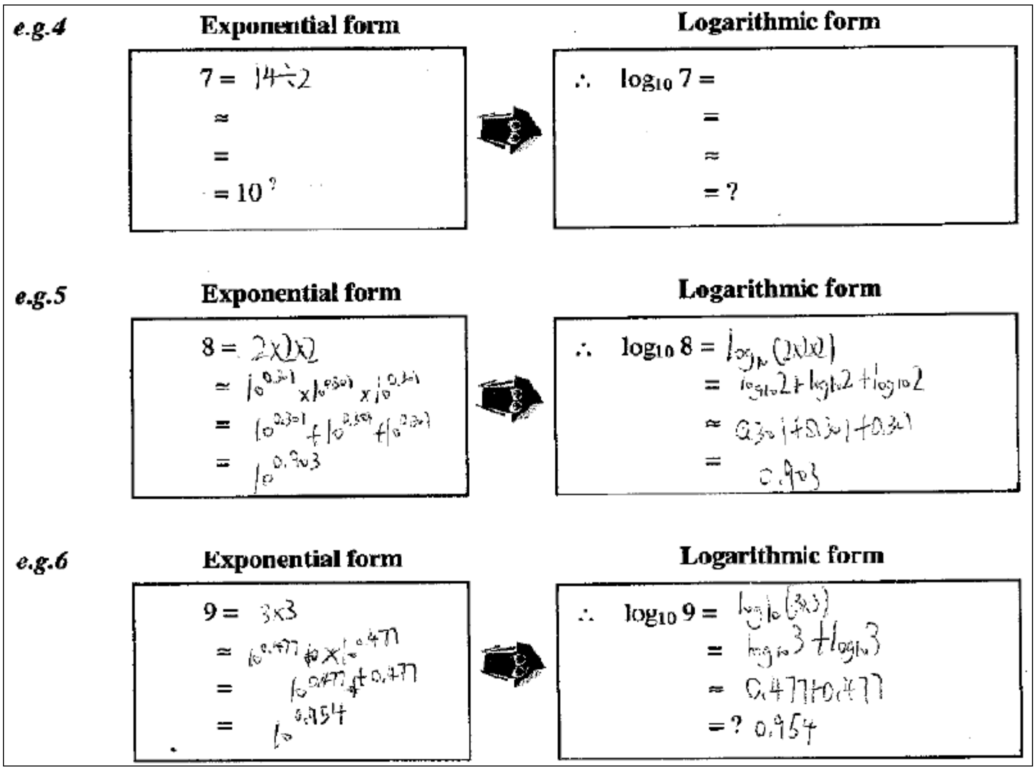


Figure 1: Students followed the guideline to find log 4, log 5 and log 6

And we can see that the students were applying the properties of logarithm (i.e. log(M × N) = logM + logN and log(M ÷ N) = logM – logN) in the worksheet, but actually those properties were not “officially” taught to them. Students had just observed the relationship between exponential form and logarithmic form and deduced the way to manipulate logarithm by themselves. After students had found the value of log 6, they were asked to find the value of log 7, log 8 and log 9. Most of them could find the values of log 8 and log 9 successfully, but they were frustrated in finding the value of log 7 (refer to Figure 2). Later they found that none of their classmate could find the value of log 7, and then I asked them a question:

“Why is it so difficult to find the value of log 7? What is the different among the number 7 and other numbers such as 6, 8 and 9?”

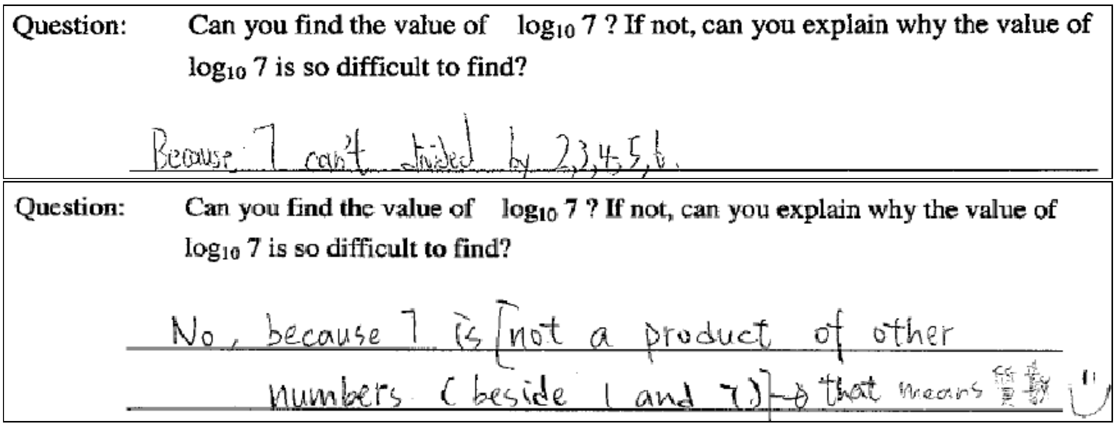
Students were asked to express their finding in the worksheet. Most of them stated that 7 could not be expressed as a product of 2, 3, 4, … and some of them could point out that 7 is a prime number (refer to Figure 3). Finally they understood why it was so difficult to find the values of log 2 and log 3, since 2 and 3 were also both prime numbers (but not log 5 since log 5 could be expressed as log(10 ÷ 2) = log 10 – log 2).


e.g.4 Exponential form
7 = 14÷2
≈
=
= 10 ?
Logarithmic form
∴ $\log_{10} 7$ =
=
≈
= ?

e.g.5 Exponential form
8 = 2x2x2
≈ $10^{0.301} \times 10^{0.301} \times 10^{0.301}$
= $10^{0.301+0.301+0.301}$
= $10^{0.903}$
Logarithmic form
∴ $\log_{10} 8$ = $\log_{10}(2\times2\times2)$
= $\log_{10}2+\log_{10}2+\log_{10}2$
≈ 0.301+0.301+0.301
= 0.903

e.g.6 Exponential form
9 = 3x3
≈ $10^{0.477} \times 10^{0.477}$
= $10^{0.477+0.477}$
= $10^{0.954}$
Logarithmic form
∴ $\log_{10} 9$ = $\log_{10}(3\times3)$
= $\log_{10}3+\log_{10}3$
≈ 0.477+0.477
=? 0.954

Figure 2: Students tried to find log 7, log 8 and log 9


Question: Can you find the value of $\log_{10} 7$? If not, can you explain why the value of $\log_{10} 7$ is so difficult to find?

Because 7 can't divided by 2,3,4,5,6.

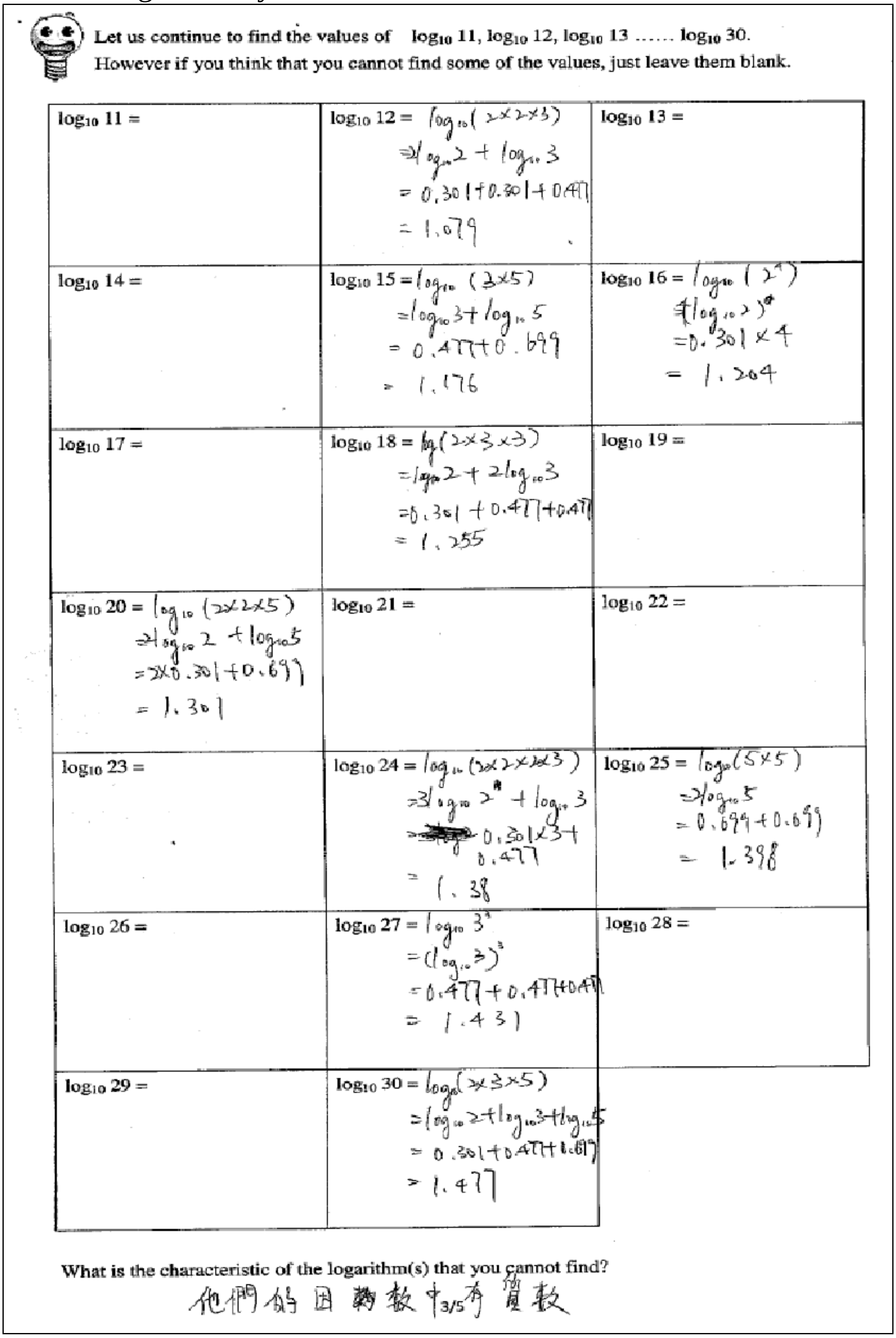
Question: Can you find the value of $\log_{10} 7$? If not, can you explain why the value of $\log_{10} 7$ is so difficult to find?

No, because 7 is not a product of other numbers (beside 1 and 7) → that means 質數 !! :)

Figure 3: Students explained why it is difficult to find log 7

After the short summary, students were asked to find the values of log 11, log 12, log 13 … to log 30. During the task, most of the students had discovered the last properties of logarithm, which is $\log M^n = n \log M$ (refer to

Figure 4). Although there were some mistakes in presentation (e.g. express log 3^3 into $(\log 3)^3$), it was glad to see that students could discover and apply the properties of logarithm by themselves.



Let us continue to find the values of $\log_{10} 11$, $\log_{10} 12$, $\log_{10} 13$ …… $\log_{10} 30$.
However if you think that you cannot find some of the values, just leave them blank.

$\log_{10} 11 =$	$\log_{10} 12 = \log_{10}(2\times2\times3)$ $= 2\log_{10}2 + \log_{10}3$ $= 0.301 + 0.301 + 0.477$ $= 1.079$	$\log_{10} 13 =$
$\log_{10} 14 =$	$\log_{10} 15 = \log_{10}(3\times5)$ $= \log_{10}3 + \log_{10}5$ $= 0.477 + 0.699$ $= 1.176$	$\log_{10} 16 = \log_{10}(2^4)$ $= (\log_{10}2)^4$ $= 0.301 \times 4$ $= 1.204$
$\log_{10} 17 =$	$\log_{10} 18 = \log(2\times3\times3)$ $= \log_{10}2 + 2\log_{10}3$ $= 0.301 + 0.477 + 0.477$ $= 1.255$	$\log_{10} 19 =$
$\log_{10} 20 = \log_{10}(2\times2\times5)$ $= 2\log_{10}2 + \log_{10}5$ $= 2\times0.301 + 0.699$ $= 1.301$	$\log_{10} 21 =$	$\log_{10} 22 =$
$\log_{10} 23 =$	$\log_{10} 24 = \log_{10}(2\times2\times2\times3)$ $= 3\log_{10}2 + \log_{10}3$ $= 0.301\times3 + 0.477$ $= 1.38$	$\log_{10} 25 = \log_{10}(5\times5)$ $= 2\log_{10}5$ $= 0.699 + 0.699$ $= 1.398$
$\log_{10} 26 =$	$\log_{10} 27 = \log_{10}3^3$ $= (\log_{10}3)^3$ $= 0.477 + 0.477 + 0.477$ $= 1.431$	$\log_{10} 28 =$
$\log_{10} 29 =$	$\log_{10} 30 = \log_{10}(2\times3\times5)$ $= \log_{10}2 + \log_{10}3 + \log_{10}5$ $= 0.301 + 0.477 + 0.699$ $= 1.477$	

What is the characteristic of the logarithm(s) that you cannot find?
他們的因數數中3/5有質數

Figure 4: Students tried to find different logarithm values

When students became familiar with the properties of logarithm, the logarithm table was introduced to the students formally and straightforwardly, and they learnt how to look for a logarithm value in the table. At the same time the bibliography of Napier and the importance of logarithm table were further introduced to the students:

> *"Napier had a great interest in astronomy, which led to his contribution to mathematics. He was involved in research that required lengthy and time consuming calculations of very large numbers. Once the idea came to him that there might be a better and simpler way to perform large number calculations, Napier focused on the issue and spent twenty years perfecting his idea. The result of this work is what we now call logarithms..."* [1]

Students were amazed why Napier could keep working on logarithm for over twenty years (they claimed that even they loved computer games, they would not spend 20 years on the games!). They were impressed by the composure and enthusiasm of Napier. Afterwards more practices had been given to the students to find different values of logarithm (refer to Figure 5).

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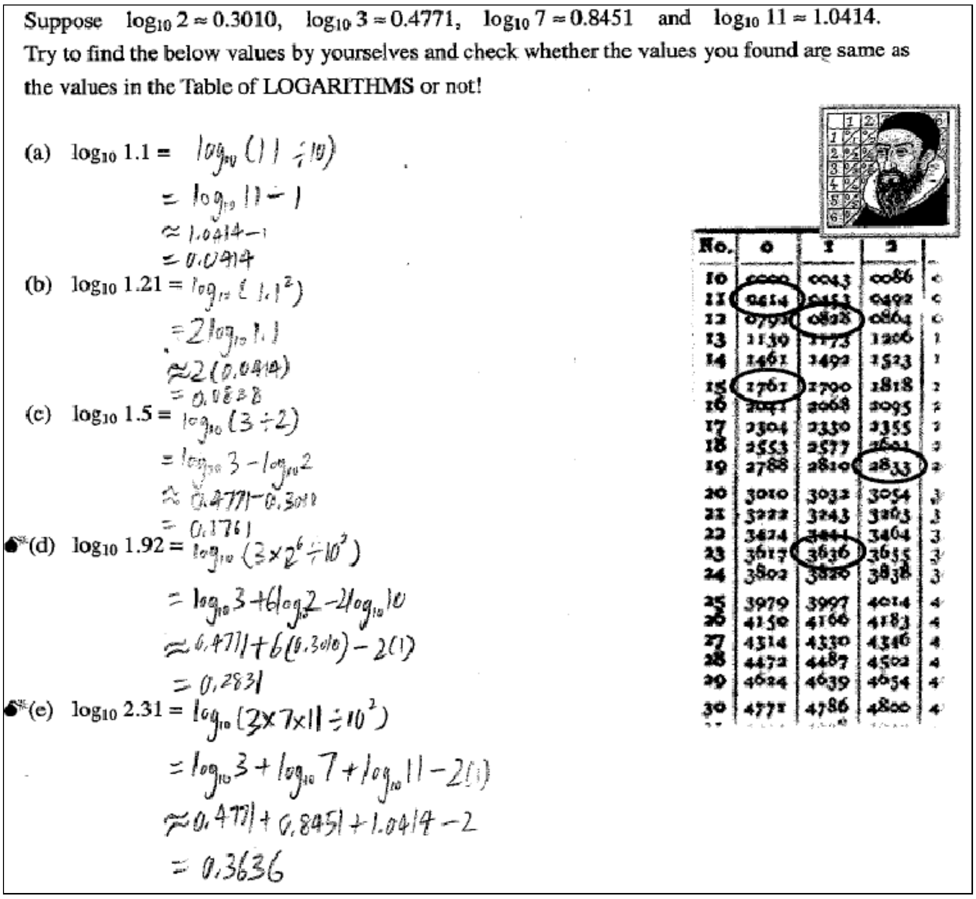
[1] Full text is available at http://math.about.com/library/weekly/blbionapier.htm .



Suppose $\log_{10} 2 \approx 0.3010$, $\log_{10} 3 \approx 0.4771$, $\log_{10} 7 \approx 0.8451$ and $\log_{10} 11 \approx 1.0414$.
Try to find the below values by yourselves and check whether the values you found are same as the values in the Table of LOGARITHMS or not!

(a) $\log_{10} 1.1 = \log_{10}(11 \div 10)$
$= \log_{10} 11 - 1$
$\approx 1.0414 - 1$
$= 0.0414$

(b) $\log_{10} 1.21 = \log_{10}(1.1^2)$
$= 2\log_{10} 1.1$
$\approx 2(0.0414)$
$= 0.0828$

(c) $\log_{10} 1.5 = \log_{10}(3 \div 2)$
$= \log_{10} 3 - \log_{10} 2$
$\approx 0.4771 - 0.3010$
$= 0.1761$

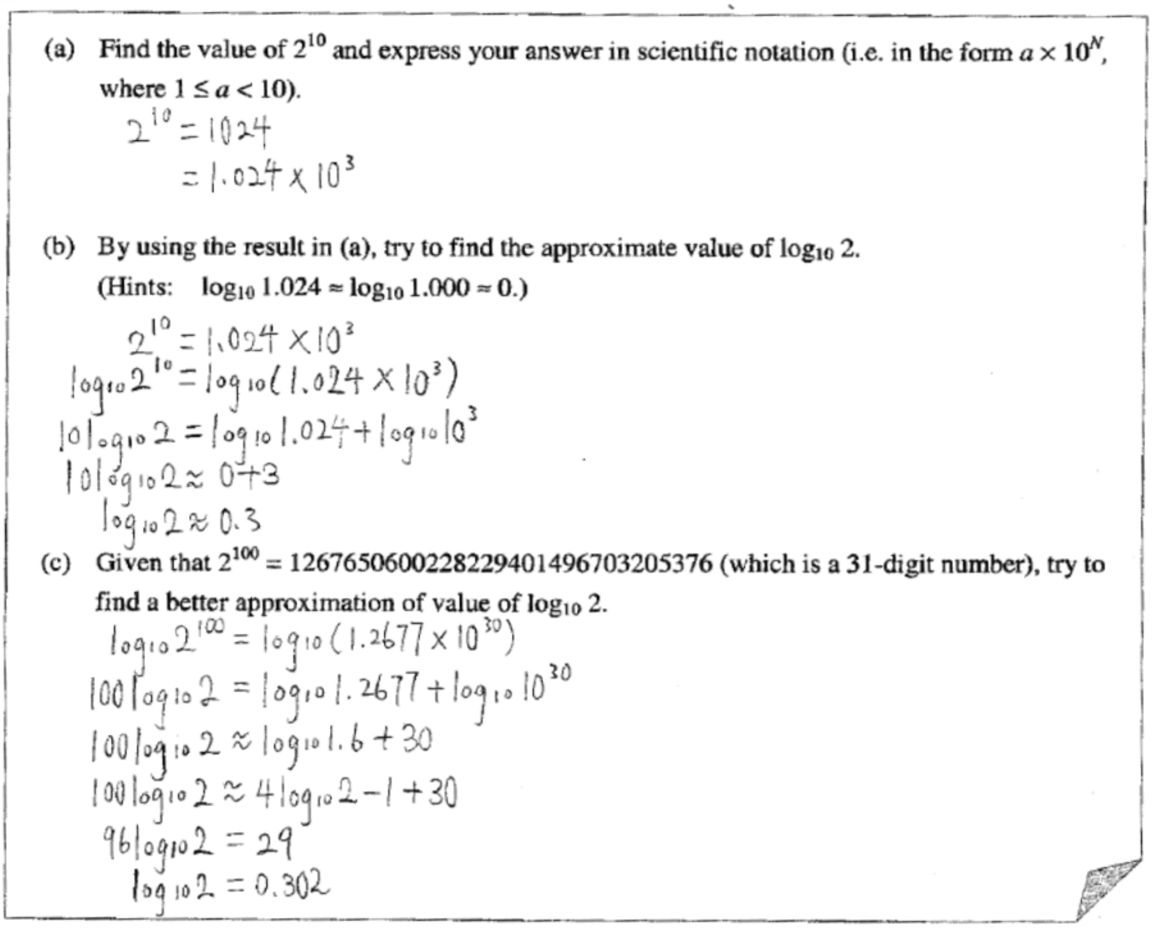
*(d) $\log_{10} 1.92 = \log_{10}(3 \times 2^6 \div 10^2)$
$= \log_{10} 3 + 6\log_{10} 2 - 2\log_{10} 10$
$\approx 0.4771 + 6(0.3010) - 2(1)$
$= 0.2831$

*(e) $\log_{10} 2.31 = \log_{10}(3 \times 7 \times 11 \div 10^2)$
$= \log_{10} 3 + \log_{10} 7 + \log_{10} 11 - 2(1)$
$\approx 0.4771 + 0.8451 + 1.0414 - 2$
$= 0.3636$

No.	0	1	2
10	0000	0043	0086
11	0414	0453	0492
12	0792	0828	0864
13	1139	1173	1206
14	1461	1492	1523
15	1761	1790	1818
16	2041	2068	2095
17	2304	2330	2355
18	2553	2577	2601
19	2788	2810	2833
20	3010	3032	3054
21	3222	3243	3263
22	3424	3444	3464
23	3617	3636	3655
24	3802	3820	3838
25	3979	3997	4014
26	4150	4166	4183
27	4314	4330	4346
28	4472	4487	4502
29	4624	4639	4654
30	4771	4786	4800

Figure 5: Work on the logarithm table by students

After finding the values of log 1.21, log 1.5, log 1.92, etc., students had more practices on the property log(M ÷ N) = logM – logN. On the other hand lots of students found that the value of log 1.92 they evaluated (which was 0.2831) was slightly different from the value stated in the logarithm table (which is 0.2833). They were frustrated since they had verified again and again and found that there were no errors in their calculation. After a short in-class discussion, some students discovered that accuracy of log 2, log 3,… given in the worksheet was not good enough (they had used calculator to clarify their finding). Then I introduced the work of another mathematician Henry Briggs, who also worked on logarithm with Napier 500 years ago. And then students were asked to follow the way of Briggs to find the value of log 2 by themselves (refer to Figure 6):



(a) Find the value of 2^{10} and express your answer in scientific notation (i.e. in the form $a \times 10^N$, where $1 \le a < 10$).

$2^{10} = 1024$
$= 1.024 \times 10^3$

(b) By using the result in (a), try to find the approximate value of $\log_{10} 2$.
(Hints: $\log_{10} 1.024 \approx \log_{10} 1.000 = 0$.)

$2^{10} = 1.024 \times 10^3$
$\log_{10} 2^{10} = \log_{10}(1.024 \times 10^3)$
$10 \log_{10} 2 = \log_{10} 1.024 + \log_{10} 10^3$
$10 \log_{10} 2 \approx 0 + 3$
$\log_{10} 2 \approx 0.3$

(c) Given that $2^{100} = 1267650600228229401496703205376$ (which is a 31-digit number), try to find a better approximation of value of $\log_{10} 2$.

$\log_{10} 2^{100} = \log_{10}(1.2677 \times 10^{30})$
$100 \log_{10} 2 = \log_{10} 1.2677 + \log_{10} 10^{30}$
$100 \log_{10} 2 \approx \log_{10} 1.6 + 30$
$100 \log_{10} 2 \approx 4 \log_{10} 2 - 1 + 30$
$96 \log_{10} 2 = 29$
$\log_{10} 2 = 0.302$

Figure 6: Students tried to find the approximation of log 2

Later students knew that Briggs had found the value of $2^{1000000000000000}$ (which is a 30102999566399-digit number) in order to find a precise approximation of log 2 = 0.30102999566398, they said that they could not believe it! They could not imagine how Briggs found the value of $2^{1000000000000000}$ 500 years ago, without using computer or calculator. On the other hand some students were inspired and continued to work on finding the values of log 3 and log 7 (refer to Figure 7):

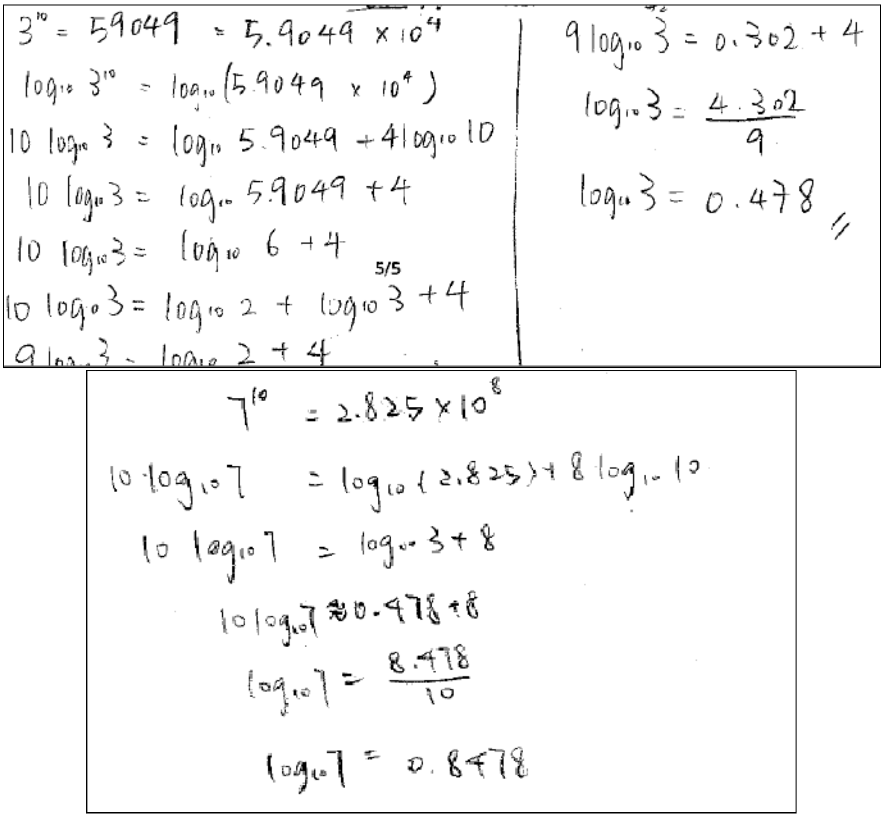


Figure 7: Students found the approximation of log 3 and log 7 respectively

After the activity students were asked to complete some exercise in the textbook. All of them could finish the exercise without major problems. Although there was still some presentation error in their works, such as $\log 27 = (\log 3)^3 = 3 \log 3$ and $\log 2^{x+1} = x + 1 \log 2$, it could see that students had learnt the properties of logarithm and applied them successfully. Throughout the learning activity, the history of development of logarithm had been integrated into the activity, but it was not the whole picture of the development of logarithm. Actually Napier had used a geometric-kinematic scheme to obtain a well definition of logarithm (Katz, 1986, 1995), but it had not been introduced in the lesson, because it involved abstract concept about manipulation of arithmetic and geometric sequences, which was quite difficult to be followed by secondary four students. In order to facilitate better understand of students about the invention and development of logarithm, the works from Briggs in *Arithmetica Logarithmica* (published in 1624) was used instead in the learning activity. According to Lam (2011), Briggs took the square of 2 to get the value of 2^2, and then took the square again to get 2^4, 2^8 and finally multiple the result by 2^2 to find the value of 2^{10}. He continued to find the values of 2^{100}, 2^{1000} and

2^{10000} by the similar way and the result was shown on the table in the fifth chapter of *Arithmetica Logarithmica* as below (refer to Figure 8):

[A] 1	0	
2	1	
4	2	1
16	4	2 First Tetrad
256	8	3
1024	10	4
10,48576	20	7
109,9511627776	40	13 Second Tetrad
12089,25819,61463	80	25
12676,50600,22823	100	31
16069,38044,25899	200	61
25822,49878,08685	400	121 Third Tetrad
66680,14432,87940	800	241
10715,08607,18618	1000	302
11481,30695,27407	2000	603
13182,04093,43051	4000	1205 Fourth Tetrad
17376,62031,93695	8000	2409
19950,63116,87912	10000	3011
	Indices	Number of places

Figure 8: The work of Briggs on finding the value of 2^{10000}

Eventually Briggs found that $2^{100000000000000}$ is a 30102999566399-digit number, and it helped him to find a nice approximation of log 2:

$$
\begin{aligned}
2^{100000000000000} &= a \times 10^{30102999566398} \quad (\text{where } 1 \le a < 10) \\
\log 2^{100000000000000} &= \log (a \times 10^{30102999566398}) \\
100000000000000 \log 2 &= \log a + \log 10^{30102999566398} \\
100000000000000 \log 2 &= \log a + 30102999566398 \\
\log 2 &= \frac{\log a}{100000000000000} + 0.30102999566398 \\
\therefore \log 2 &\approx 0.30102999566398
\end{aligned}
$$

This method only involves scientific notation and basic properties of logarithm, and they are easy to be understood by students. On the other hand, by finding the square root of 10, and then taking square root of the result again and again … we can get the values of $10^{\frac{1}{2}}$, $10^{\frac{1}{4}}$, $10^{\frac{1}{8}}$, … and use these values to find the approximation of log 2, and it is the method used to create logarithm table in 17th century (Cho, 2010). The detail is as follows:

$$10^{\frac{1}{2}} \approx 3.16228, \quad 10^{\frac{1}{4}} \approx 1.77828, \quad 10^{\frac{1}{8}} \approx 1.33352, \quad 10^{\frac{1}{16}} \approx 1.15478$$

$$10^{\frac{1}{32}} \approx 1.07461, \quad 10^{\frac{1}{64}} \approx 1.03663, \quad 10^{\frac{1}{128}} \approx 1.01815, \quad 10^{\frac{1}{256}} \approx 1.00904$$

Let $2=10^{\frac{1}{4}}\times x_1$ then we have $x_1=2\div 10^{\frac{1}{4}}\approx 1.1246$;

let $x_1=10^{\frac{1}{32}}\times x_2$ then we have $x_2=x_1\div 10^{\frac{1}{32}}\approx 1.04659$;

let $x_2=10^{\frac{1}{64}}\times x_3$ then we have $x_3=x_2\div 10^{\frac{1}{64}}\approx 1.00961$;

let $x_3=10^{\frac{1}{256}}\times x_4$ then we have $x_4=x_3\div 10^{\frac{1}{256}}\approx 1.00450$;

repeating the process to the n^{th} step and x_n will approach to 1.

Therefore $2=10^{\frac{1}{4}}\times 10^{\frac{1}{32}}\times 10^{\frac{1}{64}}\times 10^{\frac{1}{256}}\times .. =10^{\frac{1}{4}+\frac{1}{32}+\frac{1}{64}+\frac{1}{256}+...}\approx 10^{\frac{77}{256}}$.

And finally we have $\log 2\approx \log 10^{\frac{77}{256}}=\frac{77}{256}\approx 0.3$.

(Sum up more terms in the series to find a better approximation of log 2.)

All the works above had been showed to the students after the activity, and it seemed that history of mathematics could facilitate a comprehensive understanding of students in logarithm. Indeed, Fried (2008) states history can play a part in teaching and learning without altering the focus of our mathematics teaching, and it can be achieved by selecting suitable content from the history. Furthermore, Jankvist (2009) states that the manners in which the history of mathematics may be used in mathematics teaching may be classified into three major categories of approaches:

Illumination approach:	The teaching and learning is supplemented by historical information. It will be usually appeared as "isolated factual information" such as biographies, time charts and famous problems.
Modules approach:	The historical content is collected as a package which is narrowly focused on a small topic, with strong ties to the curriculum.
History-based approach:	It includes an account of historical data, a history of conceptual developments, or something in between.

Teachers can use different approaches based on their teaching objective: history-as-a-tool or history-as-a-goal (Jankvist, 2009). Obviously history-as-a-tool modules approach had been adopted in the activity. However Jankvist highlights that there is no clear-cut between different approaches and also there are various combinations of “how” and “why” to use history in mathematics education (refer to Figure 9):

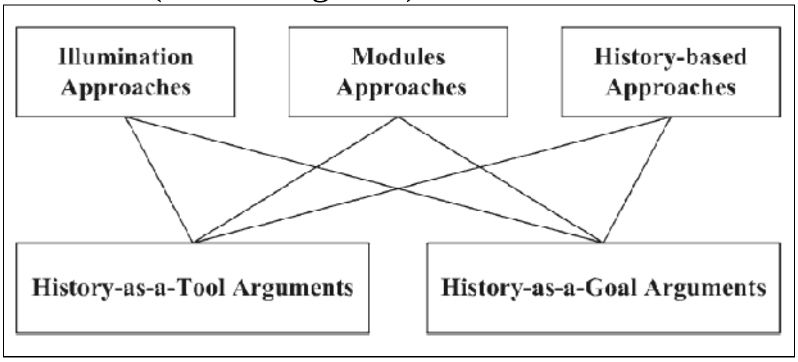
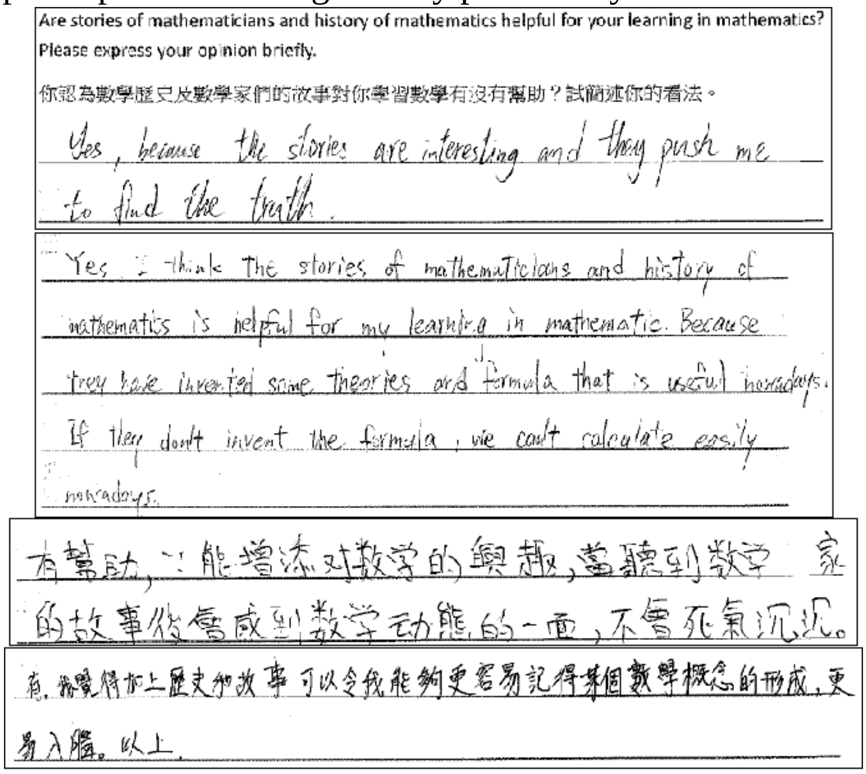


Figure 9: The six possible connections between “how” and “why” to use history in mathematics education

By using the above framework, we can obtain a clear vision and aim to apply history in mathematics teaching and learning. On the other hand, Favuel (1991) states some reasons that have been advanced for using history in mathematics education, for example:

- Helps to increase motivation for learning
- Gives mathematics a human face
- Showing pupils how concepts have developed helps their understanding
- Changes pupil perceptions of mathematics
- Comparing ancient and modern establishes value of modern techniques
- Provides opportunities for investigations
- Pupils derive comfort from realizing that they are not the only ones with problems
- Encourages quicker learners to look further
- Helps to explain the role of mathematics in society

Can the above advantages be achieved? After the lesson I had asked the students to express their opinions on integrating historical content in learning logarithm. Here are some positive feedbacks from the students (refer to Figure 10). Lots of students pointed out that story of mathematics history was interesting and help them understand the concept of some topics in mathematics (logarithm in this case). As Favuel mentioned above, history of mathematics can show how mathematics concepts have developed and hence helps the understanding of students. At the same time, the attitude of students in learning mathematics has been improved. They showed respect to the mathematicians such as Napier and Briggs and also their works on logarithm, and it motivated students to participate in learning activity proactively.



Are stories of mathematicians and history of mathematics helpful for your learning in mathematics?
Please express your opinion briefly.
你認為數學歷史及數學家們的故事對你學習數學有沒有幫助？試簡述你的看法。

Yes, because the stories are interesting and they push me to find the truth.

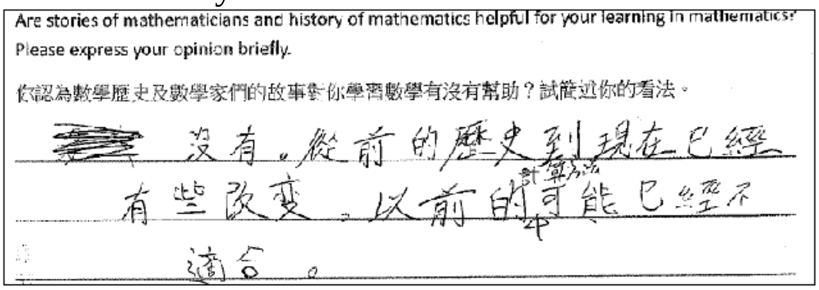
Yes I think The stories of mathematicians and history of mathematics is helpful for my learning in mathematic. Because they have invented some theories and formula that is useful nowadays. If they don't invent the formula, we can't calculate easily nowadays.

有幫助，能增添对数学的興趣，當聽到数学家的故事後會感到数学动態的一面，不會死氣沉沉。

有，我覺得加上歷史和故事可以令我能夠更容易記得某個數學概念的形成，更易入腦。以上

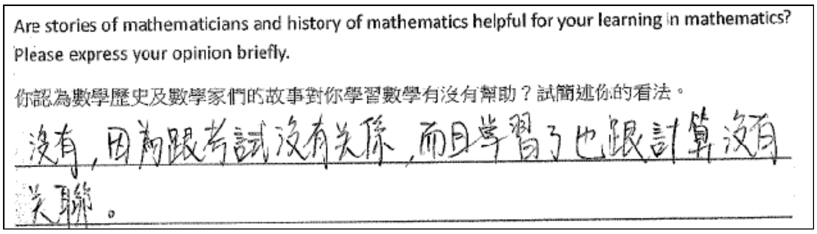
Figure 10: Some positive feedbacks from students

Meanwhile some negative feedbacks were given (refer to Figure 11). A student pointed out that the method used by Briggs in calculating logarithm was out-of-dated (Indeed Euler had designed an algorithm to find the value of log 2 (Sandifer, 2005), and it was more advanced). Few students also stated that the story of mathematics history was neither related to calculation nor examination.



Are stories of mathematicians and history of mathematics helpful for your learning in mathematics?
Please express your opinion briefly.
你認為數學歷史及數學家們的故事對你學習數學有沒有幫助？試簡述你的看法。

沒有。從前的歷史到現在已經有些改變。以前的計算方法可能已經不適合。


Are stories of mathematicians and history of mathematics helpful for your learning in mathematics? Please express your opinion briefly.

你認為數學歷史及數學家們的故事對你學習數學有沒有幫助？試簡述你的看法。

沒有，因為跟考試沒有关係，而且学習了也跟計算沒有关聯。

Figure 11: Some negative feedbacks from students

Actually the negative feedbacks above can be expected. Siu (2007) has pointed out some unfavorable factors to use history in teaching and learning mathematics, such as history of mathematics is not "mathematics", it cannot improve the student's grade in tests and examination, and students may not have enough general knowledge on culture to appreciate it. Although there are lots of concerns about using history in teaching mathematics, Siu still believes that integrating mathematics history in teaching and learning can provide a positive reinforcement on the affective side rather than cognitive side of students in mathematics. Actually Siu has already stated the similar point of view:

"In classes where history of mathematics is made use of, students like the subject more, but they do not necessarily perform better in the tests. One can argue that this may be an indication of a gap between what is taught and learnt and what is being assessed. But still, one cannot deny the possibility that students do not learn better with the addition of a historical dimension." (Siu 2007, p.276)

For instance, in 1978, Siu has already published a book to encourage students to study mathematics by referring to the history of mathematics (Siu, 1978). Moreover in 1998, International Commission on Mathematics Instruction (ICMI) has conducted a study on integrating history of mathematics in the classroom, and it was reported by Fauvel and Maanen (2000) which is by far the most comprehensive, extensive and rigorous publication in the field. Their study provides the framework and positive outcomes of integrating history in mathematics education.

Last but not least, mathematics is philosophical in its nature, but it does not mean that we must adopt a philosophical approach for every second in mathematics classroom. According to the natural way the human minds develop, Egan (1997) proposes the theory of educated mind with five kinds of understanding of the human, which are somatic, mythic, romantic, philosophical and ironic. Although the theory of educated mind are based on the study of cultural and linguistic history, Tang (2005, 2006) shows the possibilities of using mathematics history to facilitate different kinds of understanding of students in mathematics. By using the stories of Napier and Briggs, it can facilitate students' romantic understanding in logarithm. The followings are some characteristics of Romantic understanding: 1. the limits of reality and the extremes of experience; 2. transcendence within reality; 3. humanized knowledge; and 4. romantic rationality. And the central defining features of romantic understanding are the mixture of the mythic with the rational (Tang, 2006). Recall to the learning activity of logarithm, students had tried and tried again in order to find a better approximation of log 2, log 3 and so on. They learnt the concept and properties of logarithm in a humanized way, and they also experienced in pursuing limit and transcendent qualities. In the activity, the stories and works of Napier and Briggs were presented as "myth" (Napier had spent over 20 years on logarithm, Briggs had found a 30102999566399-digit number to find log 2) rather than "historical facts", and it can facilitate better romantic understanding of the students. With the adequate practices, it is believed that students can proceed to philosophical understanding (including instrumental and relational understanding) in logarithm at the same time.

Usually, there is an overt focus on calculation skills and routine learning, with drills and practice being central to the process of teaching and learning mathematics. However, mathematics is not only consisting of calculation, problem -solving and logical thinking. As Professor Siu Man Keung states that - "Mathematics is a cultural heritage". By exploring the humanistic aspects of mathematics, it is believed that both students and teachers can appreciate mathematics as a cultural-human endeavor, broaden the horizon in mathematics teaching and learning, and finally develop a positive attitude towards mathematics.

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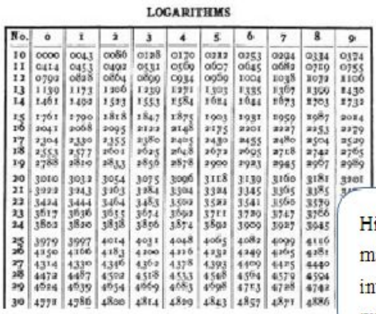
Appendix I: The worksheet used in the lesson

Po Leung Kuk Celine Ho Yam Tong College

Form 4 Mathematics

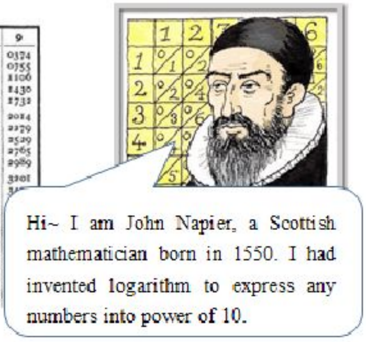
Creation of Logarithm Table

Name: ______________________________ (   )   Class: __________________



LOGARITHMS

No.	0	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	[illegible]
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	



Let us have an in-depth investigation of the above logarithmic table:

	Exponential form	Logarithmic form
1.	$10^{0} = 1$	$\log_{10} 1 = 0$
2.	$10^{0.301} \approx 2$	$\log_{10} 2 \approx 0.301$
3.	$10^{0.477} \approx 3$	$\log_{10} 3 \approx 0.477$
4.	$10^{?} \approx 4$	$\log_{10} 4 \approx ?$
5.	$10^{?} \approx 5$	$\log_{10} 5 \approx ?$
10.	$10^{1} = 10$	$\log_{10} 10 = 1$

Suppose I knew that $\log_{10} 2 \approx 0.301$ and $\log_{10} 3 \approx 0.477$, how we can find the remaining "log" values? I would like to show you the method now, however can you fill in the missing steps below?

e.g.1

Exponential form

$4 = 2 \times 2$
$\approx 10^{0.301} \times 10^{0.301}$
$= 10^{0.301+0.301}$
$= 10^{0.602}$

Logarithmic form

$\therefore \log_{10} 4 = \log_{10}(2 \times 2)$
= ____________
$\approx 0.301 + 0.301$
$= 0.602$

e.g.2

Exponential form

$5 = 10 \div 2$
$\approx 10^{1} \div 10^{0.301}$
$= 10^{1-0.301}$
$= 10^{0.699}$

Logarithmic form

$\therefore \log_{10} 5 = \log_{10}(10 \div 2)$
= ____________
$\approx 1 - 0.301$
$= 0.699$

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e.g.3

Exponential form	Logarithmic form
$6 = \square \times \square$	$\therefore \quad \log_{10} 6 = \log_{10}(\quad \times \quad)$
$\approx 10^{\square} \times 10^{\square}$	$= \square$
$= 10^{\square + \square}$	$\approx \square + \square$
$= 10^{\square}$	$= \square$

Now you should familiar how tofind the logarithm of different numbers. Try to find the values of $\log_{10} 7$, $\log_{10} 8$ and $\log_{10} 9$ below.

e.g.4

Exponential form	Logarithmic form
$7 =$	$\therefore \quad \log_{10} 7 =$
$\approx$	$=$
$=$	$\approx$
$= 10^{?}$	$= ?$

e.g.5

Exponential form	Logarithmic form
$8 =$	$\therefore \quad \log_{10} 8 =$
$\approx$	$=$
$=$	$\approx$
$=$	$=$

e.g.6

Exponential form	Logarithmic form
$9 =$	$\therefore \quad \log_{10} 9 =$
$\approx$	$=$
$=$	$\approx$
$=$	$= ?$

e.g.7

Exponential form	Logarithmic form
$10 = 10^1$	$\therefore \quad \log_{10} 10 = 1$

Question: Can you find the value of $\log_{10} 7$? If not, can you explain why the value of $\log_{10} 7$ is so difficult to find?

____

____

____

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Let us continue to find the values of $\log_{10} 11$, $\log_{10} 12$, $\log_{10} 13$ …… $\log_{10} 30$.
However if you think that you cannot find some of the values, just leave them blank.

$\log_{10} 11 =$	$\log_{10} 12 =$	$\log_{10} 13 =$
$\log_{10} 14 =$	$\log_{10} 15 =$	$\log_{10} 16 =$
$\log_{10} 17 =$	$\log_{10} 18 =$	$\log_{10} 19 =$
$\log_{10} 20 =$	$\log_{10} 21 =$	$\log_{10} 22 =$
$\log_{10} 23 =$	$\log_{10} 24 =$	$\log_{10} 25 =$
$\log_{10} 26 =$	$\log_{10} 27 =$	$\log_{10} 28 =$
$\log_{10} 29 =$	$\log_{10} 30 =$	

What is the characteristic of the logarithm(s) that you cannot find?

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Suppose $\log_{10} 2 = a$, $\log_{10} 3 = b$, $\log_{10} 7 = x$, $\log_{10} 11 = y$ and $\log_{10} 13 = z$.
Try to express the following logarithm in terms of a, b, x, y and/or z.

(a) $\log_{10} 14$

(b) $\log_{10} 15$

(c) $\log_{10} 21$

(d) $\log_{10} 22$

(e) $\log_{10} 26$

(f) $\log_{10} 28$

CHALLENGING QUESTION:

Suppose $\log_{10} 2 \approx 0.3010$, $\log_{10} 3 \approx 0.4771$, $\log_{10} 7 \approx 0.8451$ and $\log_{10} 11 \approx 1.0414$.
Try to find the below values by yourselves and check whether the values you found are same as the values in the Table of LOGARITHMS or not!

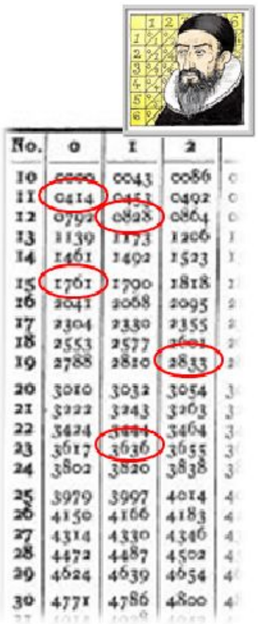
(a) $\log_{10} 1.1 =$

(b) $\log_{10} 1.21 =$

(c) $\log_{10} 1.5 =$

(d) $\log_{10} 1.92 =$

(e) $\log_{10} 2.31 =$



No.	0	1	2
10	[illegible]	0043	0086
11	0414	[illegible]	0492
12	0792	0828	0864
13	1139	1173	1206
14	1461	1492	1523
15	1761	1790	1818
16	2041	2068	2095
17	2304	2330	2355
18	2553	2577	[illegible]
19	2788	2810	2833
20	3010	3032	3054
21	3222	3243	3263
22	3424	[illegible]	3464
23	3617	3636	3655
24	3802	3820	3838
25	3979	3997	4014
26	4150	4166	4183
27	4314	4330	4346
28	4472	4487	4502
29	4624	4639	4654
30	4771	4786	4800

4/5

KNOW MORE ABOUT LOGARITHM:

Hello! I'm Henry Briggs, another mathematician which was born 1550 in England. After I had seen the work of Napier, I visited him and work on the logarithm together. Now I would like to teach you how to find the value of $\log_{10} 2$ step by step!

Are you ready? Let's GO!!!

(a) Find the value of 2^{10} and express your answer in scientific notation (i.e. in the form $a \times 10^N$, where $1 \le a < 10$).

(b) By using the result in (a), try to find the approximate value of $\log_{10} 2$.
(Hints: $\log_{10} 1.024 \approx \log_{10} 1.000 \approx 0$.)

(c) Given that $2^{100} = 1267650600228229401496703205376$ (which is a 31-digit number), try to find a better approximation of value of $\log_{10} 2$.

In order to find the best approximation of the value of $\log_{10} 2$, Briggs had found The value of $2^{100,000,000,000,000}$, which is a 30,102,999,566,399-digit number (very huge!) and finally give approximation of $\log_{10} 2 \approx 0.30102999566398$.

FINAL CHALLENGE:

Try to find a better approximation of $\log_{10} 3$ by yourselves!

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