Memoirs of a Mathematics Educator [1]

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Introduction

Teaching and learning always go hand in hand. As teachers start their career, they grow and learn from their experience in various stages of their life of teaching. Snapshots of these stages sometimes give interesting stories for sharing with colleagues and friends. My career starts in the 80's and it goes through different stages. Like all other teachers, I have learned my lessons gradually and I would like to take this opportunity to share some snapshots in my career life with readers. I organize these snapshots in two parts:

1. The very beginning: naïve or idealistic?
2. Lessons and growth in the profession.

The Very Beginning: Naïve or Unrealistic?

Snapshot 1: "I wanted to be a good mathematics teacher"

I got my first degree major in mathematics at the University of Hong Kong. Upon graduation I had a simple wish, "I want to become a good mathematics teacher." There were three aspects in this simple wish: teacher, mathematics and good. To make this wish come true, I first had to get a teaching post. Then, I needed to have a chance to teach mathematics in order to become a mathematics teacher. At early 80's, this was not difficult. Finally for the question of "good", there were three questions always pondering in my mind.

- Had I sufficient mathematics to cope with the teaching?

1 This paper is based on the third lecture in the HKAME lecture series: 蕭文強教授榮休系列講座（三）「一個數學教育工作者手記」presented on July 2, 2005.

- Did I like mathematics?
- Did I know how to teach?

I am not sure whether this is a naïve wish or an unrealistic wish. However, I still glad that I have had made such a wish since I started teaching. It motivates me to move along and keep learning.

Snapshot 2: My students in my first year of teaching

I found a teaching post in a new school at Tuen Mun. In those days Tuen Mun was newly developed and many housing estates were still under construction. To support the projected population growth in the near future, schools were built rapidly. As a result, in order to fill up the places in the school, we had students coming from other areas far away from Tuen Mun. Their academic results were below average. They made many mistakes in their classwork, homework and tests. I had never seen so many mistakes in my life. They did not like to do their homework. I worried for their examination results more than they did. I was confused by their behavior and could not understand them.

There were a few instances that helped me learn some very important lessons in my career:

The first case was the lesson observed by my supervisor Dr. Matthew Linton when I studied for the Postgraduate Certificate of Education Programme. It was a secondary lesson on the topic of the sum of an arithmetic progression. I started the lesson first by asking the students for the sum of "1 + 2 + 3 + … + 100", followed with the story of Gauss. Then I derived the formula $S(n) = (T(1) + T(\mathrm{n})) \times n \div 2$. After this, I gave examples of the application of the formula and I gave the students some time for classwork. Everything seemed to be very obvious and straightforward to me till I invited a student to answer a question. I asked her to put some numbers into the formula to give an answer. She failed to do it despite all my effort of giving hints. Eventually I had to let another student to answer the lesson. I felt very frustrated though Dr. Linton liked the development of the content of my lesson. He also gave a remark that

kept me thinking for a long time. He said that the failure in helping the girl to answer the question helped him understand how weak the students were.

Dr. Linton's remark helps me realize that I had been put focus on what I could give all the time but missing an important part in teaching, i.e. to understand my students from their perspectives. This is also a phenomenon I often notice in many student teachers. They put in great effort to prepare their lessons and present their work in a sensible and logical way. This is of course important. Nonetheless, this is only a part of the teaching. The teaching will never be completed till the teacher understands the students and takes their understanding into consideration in their planning and actual teaching.

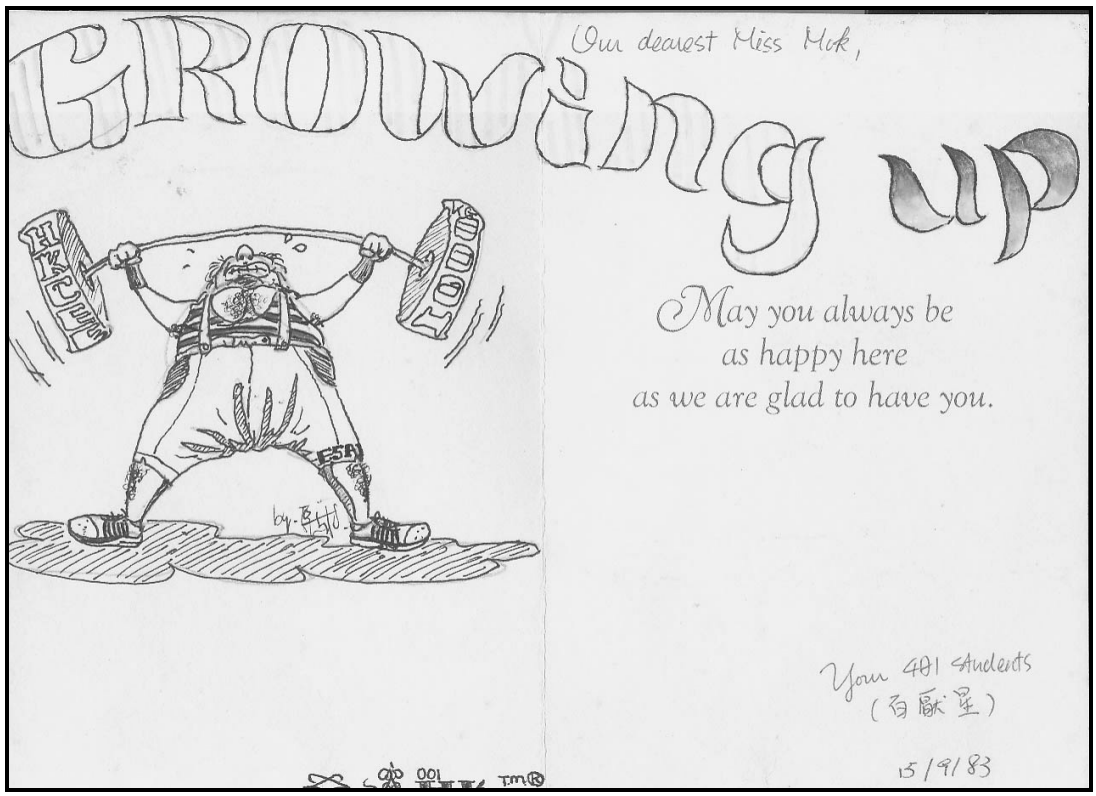

Figure 1: The invitation card

Another instance happened when I was about to change school and said my farewell to my students. A student in the class made a sulky remark, "You do not like to teach us." Even though I had good rapport with the class, the remark was unfortunately true to a certain extent deep inside my heart at that moment. "Shame on me." I had to confess that I at that moment wanted to

teach students with better academic results and had not appreciated enough the students in my first year of teaching. One year later, I received their invitation card (figure 1) for a picnic. It was a beautiful card they drew themselves. The picture was a great fellow under the pressure of the HKCEE exam, trying very hard to lift a weight. On top of it was the words "growing up".

Indeed, one needs to grow up, both the students and the teacher. Teachers can never be good if they work only for the examination results. They will become captive under the system and lose many treasures and appreciation in their teaching life.

Lessons and Growth in the Profession

Snapshot 3: What I learned is not enough

My second job was in a Band One school where I spent 7 years. I had to teach mathematics for several examination syllabuses: A-Level, Higher-Level, GCE O-Level, HKCE level. On top of this, I had to teach mathematics for two classes F.6 Biology and F.6 General Studies with self-designed curriculum tailored for the students. Occasionally the colleague of F.6 Psychology will invite me to teach her class a lesson on inferential statistics. I also had a chance to teach F.1 and F.3. Besides mathematics, I also had to teach F.4 Chemistry as a part of my scheduled workload. Furthermore, there was involvement in extracurricular activities including: Mathematics Club, Competition (Statistical Project) and Science Club. The students were keen and they might bring to you any mathematics they came across and invite an explanation or discussion. It might be a game, a newspaper cutting, a question from a test paper of unknown source or a follow up after listening to a public lecture. I also successfully got the permission to include mathematics project as a part of the term assessment in my 2 years of F. 1 teaching.[2] In the same period, I completed my Master in Education via part-time study.

The years are in fact very demanding and challenging. The experiences

2 Mok, I. A. C. 1995. A Mathematics Project Experience. *Edumath*, Dec, 1995, 30–33.

with my students and my job have driven me to work hard, learning more mathematics, and exploring new approaches. I learn that I will never learn enough in my profession.

Snapshot 4: Proof: Why bother?

> *Proof, in its best instances, increases understanding by revealing the heart of the matter. Proof suggests new mathematics. The novice who studies proofs gets closer to the creation of new mathematics.* (Davis and Hersh)[3]

In my teaching life in secondary schools I had to teach high form mathematics most of the time. Proof became an authority in my choice for explanation as well as a handy tool in various mathematical investigations. I always tried my best to lead students see the meaning behind individual steps and statements whenever there was an opportunity. There was an interesting observation in a proof for "there is an infinite number of prime numbers" that I quote below.[4]

> Suppose that there are finitely many prime, say, n_1 , n_2 , n_3 , … , n_k . Now consider the number $n_1 n_2 n_3 \ldots n_k + 1$. None of the existing prime numbers n_1 , n_2 , n_3 , … , n_k is a factor of $n_1 n_2 n_3 \ldots n_k + 1$.
> Therefore, $n_1 n_2 n_3 \ldots n_k + 1$ must also be a prime. This is absurd. Therefore, the initial assumption, that there are finitely many prime, must be wrong. (Q.E.D.)

Finishing this proof, another question comes up. In the writing of the proof, a number appears in focus: $n_1 n_2 n_3 \ldots n_k + 1$. Is this number a prime? Interested readers may give it a trial.

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3 Davis, P.J. and Hersh, R. 1981. *The Mathematical Experience*, p.151.

4 Mok, I.A.C. 1997. Proof: Why bother? *Edumath*, 5, December, pp.67–69.

2	$2 + 1 = 3$	Yes
2, 3	$2 \times 3 + 1 = 7$	Yes
2, 3, 5	$2 \times 3 \times 5 + 1 = 31$	Yes
2, 3, 5, 7	$2 \times 3 \times 5 \times 7 + 1 = 211$	Yes
2, 3, 5, 7, 11	$2 \times 3 \times 5 \times 7 \times 11 + 1 = 2311$	?
2, 3, 5, 7, 11, 13	$2 \times 3 \times 5 \times 7 \times 11 \times 13 + 1 = 30031$	?

Snapshot 5: Is mathematics truly precise and concise?

Very often people say that mathematics is precise and concise. Then they assume that there was no ambiguity in the language of mathematics. However, this is not the case. William Wynne Willson stated "Five types of ambiguity" in the 1994 Presidential Address that,

> (This is) *convenient for the sophisticated mathematician, but can be a stumbling block to learners.*[5]

Willson mentioned some examples that may be found in our teaching. For example, when we write two things next to one another, we adhere many different meaning to the symbols:

- "*ab*" means "*a* times *b*" but what happen to a = 3 and b = $\frac{1}{2}$?
- *AB*
- d*x*
- What is the meaning for "–" in 6 – (–(–3)) ? The symbol is often read as "minus" but it may mean: a binary operation (subtraction), a unary operation (taking the additive inverse of), or a label distinguishing negative 3 from positive 3.

5 The Mathematical Gazette, v. 78, n. 482, pp.174–187.

Snapshot 6: A boy's remark after the entrance examination of a primary school

> *Jack had just come out from the examination hall of a primary entrance examination. He said, "They are only additions. I can do all the problems. They put the '+' in a slanting position. I have corrected them."*[6]

This is a real story told by a parent to an old friend. Jack's result in this examination probably was not very desirable. However, we cannot help saying that he is truly smart. Students' thinking is always valuable.

Snapshot 7: Book recommendations

Keep learning is very important in the professional growth of a teacher. Learning comes from all forms of activities: observation, investigation, reading, sharing with friends, seminars, attending courses and even teaching itself. Reading is a good habit. There are a few books written by Professor Siu Man Keung that I find resourceful and enjoy sharing with my students. They are listed below:

- 《為甚麼要學習數學？—數學發展史給我們的啟發》蕭文強
- 《概率萬花筒》蕭文強、林建
- 《1, 2, 3 …, 以外》蕭文強
- 《數學証明》蕭文強

Snapshot 8: Does research help?

In the 21st century teachers face many new challenges. In addition to preparation of daily lessons and pastoral care for students, they need to catch up with different kinds of issues and initiatives advocated by curriculum reform activities. Sometimes there is a need for research. Does research help? A

6 Mok, I.A.C. (2009). *Learning of algebra: Inspiration from students' understanding of the distributive law*, p.15. Hong Kong: Hong Kong Association for Mathematics Education.

simple answer is “Yes.” There are many issues in teaching, learning and curriculum that cannot be resolved in a short time. Research is not a panacea but it is an opportune to study a question systematically. The dissemination of the results is in fact an important part of professional sharing.

Here, I take the opportunity here to share a few reports that I have a chance to work with other colleagues in the field.

Wong, N.Y., Lam, C.C., Wong, K.M., Leung, F.K.S., Mok, I.A.C. (2003). How do Hong Kong students think about mathematics. *Edumath*, 16, 7–21.

Wong, N.Y., Lam, C.C., Wong, K.M., Leung, F.K.S. , Mok, I.A.C. (2002). How do Hong Kong students think about the mathematics curriculum and teaching. *Edumath*, 15, 2–16.

Wong, N.Y., Lam, C.C., Wong, K.M., Leung, F.K.S., Mok, I.A.C., (1999). *An analysis of the views of various sectors on the mathematics curriculum: Final report*. http://cd1.edb.hkedcity.net/cd/maths/en/ref_res/document/Research2.htm

Note: The research was commissioned to the Chinese University of Hong Kong by the Education Department in response to the need for a review and possible reform in the mathematics curriculum in Hong Kong. Led by Wong Ngai-ying, we, the research team, studied the views of the key stakeholders including students, parents, teachers, university lecturers, curriculum planners and human resources personnel in the commercial sector. The research was commenced in April 1998 and completed in June 1999. Many teachers in my experience are interested in studying students’ perception in various contexts. The report is among the most frequent items that I recommend to my students who are interested in this area.

Fung, C.I. (HKIEd) & Mok, I.A.C. (1997). Primary school arithmetic in Hong Kong: Shall we discard the procedural paradigm? *Education Journal*, 25(2): 63–80.

Note: The interest of this study came from an observation of a primary student’s quote of the distributive property in an interview in 90’s. Fung Chun-ip and I scrutinized the popular local textbooks to study how the basic arithmetic operations were presented in textbooks. Via a framework of procedural-conceptual-proceptual, we discuss issues we

found in the textbooks as well as the teaching of arithmetic in the local context.

Mok, I.A.C., Johnson, D.C. (2000). Reasoning algebraically with IT: A cognitve perspective. *Mathematics Education Research Journal*, 12(3), 286–302.

Note: This was the first teaching intervention project I carried out in Hong Kong after I finished my PhD. At that moment, the Cognitive Acceleration in Mathematics Education (CAME) project has just reported promising results in its longitudinal study in the UK and it was also the time at which we were about to consider how to use IT in teaching mathematics. With the guidance of David Johnson, I applied the theoretical framework of CAME in the context of graphical calculators in a small-scale teaching intervention project. The study was carried out by a secondary teacher to a class of secondary three and a class secondary six. In the paper we reported lesson scenarios to demonstrate how the learning with respect to the paradigm of constructivism and social constructivism could be constructed via interplay between the teacher, the students, the tasks and the graphical calculators.

Mok, I.A.C., Morris, P., (2001). The Metamorphosis of the 'Virtuoso': Pedagogic patterns in Hong Kong primary mathematics classrooms. *Teaching and Teacher Education: An International Journal of Research and Studies*, 17(4), 455–468.

Note: The Target Oriented Curriculum advocated by the local government received active views and comments (both positive and negative) from various educational sectors in the local contexts. The paper analysis reported the findings from the TOC evaluation project led by Paul Morris. Results provided evidences that the reform had brought about changes in the style and practice in primary math lessons and the image of the teacher's 'virtuoso' image reported in the 80's has gone through changes.

Lee, A.M.S., Wong, K.L., Mok I.A.C. (2003). Implementation of ICT in the Mathematics Teaching in Hong Kong: Teacher Conceptions and Evolving Classroom Practices. *Information Technology, Education and Society*, 4(1), pp.117–133.

Note: This study reports on an investigation of the reform initiatives in the integration of ICT in the mathematics curriculum introduced by the Hong Kong Government and the changes effected from the teacher's perspective. Data was drawn from two sources: (a) documents such as syllabuses, teaching packages, contents of workshops and seminars; and (b) interviews with teachers who are experienced users of ICT in mathematics teaching.

Mok, I.A.C., Cai, J., Fung, A.T.F. (2008). Missing learning opportunities in classroom instruction: Evidence from an analysis of a well-structured lesson on comparing fractions. *The Mathematics Educator*, Vol.11, No.1/2, 111−126.

Note: There is a dilemma between the teacher's guidance in the lesson and the opportunities for students expressing their views. Too little guidance may cause chaos, whereas too much control will need to missing opportunities. A teacher needs to be aware of the dilemma, hence enable them to reflect upon their practices and make fine adjustment in their daily teaching. The authors in this paper present the analysis of a lesson and discuss the two sides of the coin.

Mok, I.A.C. (2009). In search of an exemplary mathematics lesson in Hong Kong: An algebra lesson on factorization of polynomials. *Zentralblatt fuer Didaktik der Mathematik (ZDM Mathematics Education)*. 41, 319−332.

Mok, I.A.C. (2006). Shedding Light on the East Asian Learner Paradox: Reconstructing Student-centredness in a Shanghai Classroom. *Asia Pacific Journal of Education*, 26(2), 131−142.

Note: I have the opportunity to join The Learner's Perspective Study (LPS) since 2000. The project gives me a lot of opportunities to learn from international partners in issues of research methods, mathematics teaching and learning in both the local context and the context of international comparison. The Learner's Perspective Study (http://extranet.edfac.unimelb.edu.au/DSME/lps/) examines the patterns of participation in competently-taught eighth grade mathematics classrooms in sixteen countries in a more integrated and comprehensive fashion than has been attempted in previous international studies. The aim of our research is to document not just the obvious social events that

might be recorded on a videotape, but also the participants' construal of those events, including their memories, feelings, and the mathematical and social meanings and practices which arose as a consequence of those events. The Hong Kong component led by Frederick Leung collected data for Hong Kong and Shanghai. My work is mostly on the data from the two cities. The two papers present analysis of the lessons and a triangulation between the views of a researcher, the teachers and the students.

Epilogue

My career gradually develops and grows over the years: from a mathematics student in my undergraduate study, to a mathematics teacher, then finally a teacher for mathematics teachers. As we move along, we learn more about the nature of the subject and the nature of the learning of the subject. The life of a mathematics teacher is indeed forever growing. It is essential for teachers to always keep an interest in the subject. To end, I would like to share with readers this message:

> *In reality, the appreciation of the power of mathematics never comes without effort. We need to keep a curiosity in the world around us. In searching for an understanding of the world around us, we formulate more problems than solutions. In searching for solutions, we use mathematics and inevitably realize that we have never learnt enough mathematics. Instead of saying how important mathematics is, a more effective way to be aware that mathematics is a lived object in our world is probably to have a curiosity in the world around us and the perseverance to satisfy our curiosity.* [7]

7 Mok, I.A.C. 2002. Reflections on the aims and objectives of teaching mathematics: A word to mathematics teachers at the beginning of the semester. *Edumath* 15, 45–47.