How do Hong Kong Students think about mathematics

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Introduction

Numerous studies revealed how students think about mathematics have direct influence on how they learn mathematics and how they tackle mathematics problems (Cobb, 1985; Crawford, Gordon, Nicholas & Prosser, 1998; McLeod, 1992; Pehkonen & Törner, 1998; Schoenfeld, 1983; Underhill, 1988). In Hong Kong, a research team comprise the first three authors of this paper started off the investigation of students' and teachers' conceptions of mathematics and of mathematics learning since 1996 and arrived at fruitful results (Lam, Wong, & Wong, 1999; Wong, 2000, 2001a, 2001b, 2002; Wong, Lam, & Wong, 1998, 1999; Wong, Marton, Wong, & Lam, 2002; see also Wong et al, 2002; Wong & Sun, 2002). In sum, students associated mathematics with its terminology and content, and that mathematics was often perceived as a set of rules. Wider aspects of mathematics such as visual sense and decision making were only seen as tangential to mathematics. In particular, they were not perceived as "calculable." However, students did recognise mathematics as closely related to thinking. The task of mathematics problem solving is thus, in essence, the search for keywords and routines. Some students even tried to approach a mathematics problem by identify which topic it lies in and trying all the related formulas they have learned. As students' conceptions of

mathematics are largely shaped by their experiences in the mathematics classroom, teachers' conceptions of mathematics, which are reported elsewhere to resemble basically those of the students, were investigated in these studies too.

In the holistic review of the Hong Kong mathematics curriculum, a research was commissioned to the authors to investigate different stake-holders' view on the mathematics curriculum(*). Apart from other results (see Wong et al, 1999, for details), students' conceptions of mathematics were again investigated. Fifteen groups of students, in groups of four, from twelve schools were invited to attend a semi-structured interview. Both boys and girls were included except for students from single-sexed schools. Details of their characteristics are as follows.

	Number of students				
Academic ability	Primary 3	Primary 6	Secondary 3	Secondary 4	Secondary 6
High	4	4	4	4	4
Average	4	4	4	4	4
Low	4	4	4	4	4

Before the interview, the students were requested to respond to three open-ended questions. They are

(a) Mathematics is …
(b) Mathematics learning is …
(c) The mathematics class is …

In this paper, we would like to report on the results obtained with these open-ended questions (see Wong, 1993, 1996 for earlier use of these questions).

Open-ended questions

The three open-ended questions probed into the students' perception of mathematics, mathematics learning and mathematics classroom respectively. The students' written responses were analysed and the results are given below.

(*) Authors' Note: The research was commissioned by the Education Department, Hong Kong.

Conception of mathematics

Primary Three

Most Primary 3 students said that mathematics was an interesting subject, but they were not specific about what they found interesting. Some students focused on learning in general:

> Mathematics is about acquiring learning a lot of knowledge. (Sz-P3-1-L)(#)
>
> Mathematics is a subject. (Sz-P3-4-L)

One Primary 3 student focused on subject specific learning. This response also concerned the intrinsic nature of mathematics:

> Mathematics is +, −, ×, ÷ . (Sze-P3-3-L)

Primary Six

New categories emerged in Primary 6 students' responses. An outstanding one was the appreciation of the functional aspect of mathematics. We got responses like,

> Mathematics helps people think. (Md-P6-3-H)
>
> If we do not learn mathematics, we will not know how to do computation. (Tg-P6-1-M)
>
> Mathematics trains our mind to be more active. (Tg-P6-2-M)
>
> Mathematics trains our mind. (Tg-P6-3-M)
>
> Mathematics exercises our brain. (Tg-P6-4-M)
>
> Mathematics helps us record or calculate quantity. (Sz-P6-2-L)

It could be seen that many students saw doing mathematics as a training of the mind.

(#) Authors' Note: Sch-G-n-S refers to quotation n in the interview with a student at grade level G in school Sch with academic standard S (L for low, M for medium, H for high).

Primary 6 students used “interesting”, “difficult” and “challenging” to describe their perception of mathematics. When compared with Primary 3 student responses, there were fewer responses that described mathematics as “interesting”. On the other hand, these results showed that students of Primary 6 had already found mathematics to be a difficult and challenging subject. Here are some of their responses:

> Mathematics is interesting, but it is a relatively difficult subject. (Md-P6-1-H)
>
> Mathematics is interesting, but some topics are relatively difficult. (Md-P6-2-H)
>
> Mathematics is a challenge; there is a sense of achievement from being able to solve a problem quickly and accurately. (Sz-P6-1-L)
>
> Mathematics is a challenging and difficult subject. (Sz-P6-4-L)

Compared with Primary 3 student responses, there was more focus on learning in general:

> Mathematics is knowledge that has a lot of variations. (Md-P6-4-H)
>
> Mathematics is something we must learn. (Tg-P6-1-M)
>
> Mathematics is a subject. (Tg-P6-3,4-L)

Although not a strong theme, it is clear that some students were looking at the intrinsic nature of mathematics but most of the responses reflected that mathematics was perceived as something calculable, such as,

> Mathematics is made up of numbers. (Sz-P6-2-L)
>
> Mathematics is made up of numbers and then we do computations based on those numbers. (Sz-P6-4-L)

Secondary Three

More students at this level started to look at the intrinsic nature of mathematics. This is an important theme in this group of responses:

> Mathematics is a subject that emphasises understanding rather than memorisation and requires a lot of practice. (Wg-S3-3-H)
>
> Mathematics is what we use to do computation on numbers. (Le-S3-1-M)
>
> Mathematics requires strong powers of comprehension in order to understand it. (Lee-S3-4-M)
>
> Mathematics is about computation of numbers. (Lo-S3-1-L)
>
> Mathematics is a subject that requires thinking. (Lo-S3-3-L)
>
> Mathematics is a subject that requires exercising of the brain. (Lo-S3-4-L)

Students who looked at the intrinsic nature of mathematics focused mostly on the requirement of understanding, thinking and practice in the subject.

A newly emerged theme was that students started to look at the applications and uses of mathematics:

> Mathematics is something that has a lot of applications in daily life. (Wg-S3-1-H)
>
> Mathematics is something that is indispensable and it has a lot of applications in engineering. (Wg-S3-2-H)

There was a further decrease in the number of responses that perceived mathematics as “interesting”. No students perceived mathematics as “difficult”. On the other hand, results showed that Secondary 3 students focused more on the learning aspect of mathematics than students of the two primary levels.

Secondary Four

Compared with Primary 3, Primary 6 and Secondary 3, there was stronger focus on learning. We have,

> Mathematics is a discipline. (SP-S4-1-H)

> Mathematics consists of very deep knowledge. (SP-S4-2-H)

Compared with Secondary 3 students' responses, Secondary 4 students also emphasised the intrinsic nature of mathematics. Secondary 4 students' responses showed that, compared to Secondary 3 students, they looked much more deeply at the intrinsic nature of the subject. Here are some of the responses:

> Mathematics makes use of quantitative data and formulae to work out unknown solutions through computation. (SP-S4-3-H)
>
> Mathematics is a subject that translates abstract principles into concrete computation. (Mk-S4-2-M)
>
> Mathematics is a collection of formulae. (Mk-S4-3-M)
>
> Mathematics is a subject that requires a good solid foundation in basics and knowledge of this subject is accumulative. (Mk-S4-4-M)
>
> Mathematics is about computation and computation tools. (Sm-S4-2-L)
>
> Mathematics is a tool for logical thinking and a tool for data organisation and analysis. (Sm-S4-3-L)

The functional aspect of mathematics first appeared in Primary 6 student responses and appeared again in these Secondary 4 responses:

> Mathematics is a subject that trains and exercises our brain. (Mk-S4-1-M)
>
> Mathematics is a subject that can promote intellectual development. (Se-S4-1-L)
>
> Mathematics can promote thinking. (Sm-S4-4-L)

Again, the most prominent functional aspect was training in thinking.

Compared with Secondary 3 students' responses, there was a slight increase in the number of students who described mathematics as "interesting".

Secondary 3 students started to look at the applications of mathematics. This theme appeared again in this group of responses. At the Secondary 4 level, the number of students who focused on the applications of the subject was slightly higher than that of Secondary 3 students.

Secondary Six

In this group of responses, there was an obvious drop in the focus on learning. Most Secondary 6 students focused on the intrinsic nature of mathematics, but the number of students who did so was fewer than those in Secondary 3 and Secondary 4. No student described mathematics as "interesting". There were students who described mathematics as "difficult" and "challenging". A new category which emerged was that mathematics is something "abstract."

Conception of mathematics learning

Primary Three

As in the previous case, most Primary 3 students perceived mathematics learning as interesting. One student perceived this learning as "acquiring more knowledge in mathematics" (Tg-P3-4-M). Another student focused on the intrinsic nature of mathematics and saw mathematics learning as something involving computation (Sz-P3-1-L). One student saw mathematics learning as a way of "getting good (academic) results" (Sz-P3-4-L).

Primary Six

Most Primary 6 students perceived mathematics learning in terms of the content of this learning:

> Mathematics learning is about enhancing our computation ability. (Md-P6-1-H)
>
> Mathematics learning shows us how to approach mathematical problems. (Tg-P6-2-M)
>
> Mathematics learning teaches us how to use different methods to compute (Sz-P6-2-L)
>
> Mathematics learning teaches us different computational methods.

(Sz-P6-4-L)

Most Primary 6 students were equally concerned about the functions of mathematics learning. We have:

> Mathematics learning promotes our thinking. (Md-P6-2-H)
>
> Mathematics learning prepares us for society. (Md-P6-3-H)
>
> Mathematics learning makes us use our brain and think. (Tg-P6-3,4-M)

Some students saw mathematics learning as "knowing" and "understanding":

> Mathematics learning reveals to us the secret of numbers. (Md-P6-2-H)
>
> Mathematics learning gives us a better understanding of mathematics. (Md-P6-4-H)

Some students looked at mathematics learning in terms of its applications:

> Mathematics learning enables us to know how we can apply it in our everyday life. (Tg-P6-1-M)
>
> Though mathematics does not have a lot of uses, sometimes it is useful. (Tg-P6-2-M)

While most Primary 3 students perceived mathematics learning as interesting, only one Primary 6 student found mathematics learning interesting. This is an indication of a drop in the interest level.

Secondary Three

Most Secondary 3 students perceived mathematics learning in terms of the various components of this learning. These components included "understanding", "putting in a lot of hard work", "consulting people", "doing a lot of exercises" and "thinking":

> Mathematics learning emphasises understanding. (Wg-S3-1-H)
>
> Mathematics learning involves a lot of hard work. When we come to something we do not understand, we have to consult people and follow it up with a lot of exercises. (Wg-S3-3-H)
>
> Mathematics learning involves a lot of thinking before coming to an answer or a way of dealing with a problem. (Wg-S3-4-H)
>
> Mathematics learning is thinking. (Lo-S3-2-L)

Some Secondary 3 students perceived mathematics learning in terms of its functions, in particular, the function of training a person's thinking abilities:

> Mathematics learning is a training of the mind. (Wg-S3-1-H)
>
> Mathematics learning trains people's thinking and comprehension abilities. (Wg-S3-4-M)
>
> Mathematics learning prepares a person for the future. (Lo-S3-3-L)

Some students were concerned about the applicability of knowledge gained in mathematics learning.

> Mathematics learning can be used in daily life. (Le-S3-1-M)
>
> Mathematics learning makes many things convenient in our daily life. (Le-S3-2-M)

Again, only one student perceived mathematics learning as interesting.

Secondary Four

Most Secondary 4 students were concerned about the various aspects of mathematics learning. They perceived mathematics learning as "understanding", "an opportunity to accumulate experience", "requiring a high degree of discipline", and "broadening the scope of thinking."

In addition, many Secondary 4 students looked on mathematics learning in terms of its content:

> Mathematics learning helps us understand mathematical principles. (SP-S4-1-H)
>
> Mathematics learning is acquiring the techniques of solving mathematical problems. (Mk-S4-4-M)
>
> Mathematics learning is learning methods and applications in mathematics. (Sm-S4-2-L)

Some students looked at mathematics learning in terms of its functions in developing logical thinking, analytical power, and independent thinking. One student saw mathematics learning in terms of its applicability in daily life: “Mathematics learning is for applications in daily life” (SP-S4-2-H). Only one student perceived mathematics learning as interesting.

Secondary Six

Most Secondary 6 students were concerned about the functional aspect of mathematics learning, with particular focus on the cultivation of thinking abilities. We have,

> Mathematics learning trains logical thinking. (Yg-S6-1-H)
>
> Mathematics learning trains and develops thinking and interest. (Yg-S6-3-H)
>
> Mathematics learning is a method of cultivating thinking abilities. (Yg-S6-4-H)
>
> Mathematics learning trains our thinking. (Hn-S6-2-M)
>
> Mathematics learning exercises the brain. (Hn-S6-4-M)
>
> Mathematics learning exercises the brain and trains our thinking. (Cn-S6-2-L)

In addition, many students saw mathematics learning in terms of its various aspects.

> Mathematics learning is a process of analysis and comprehension. (Hn-S6-3-M)

> The process of mathematics learning takes time. (Hn-S6-4-M)
>
> Mathematics learning does not require a lot of memorisation. (Cn-S6-4-L)

One student looked at mathematics learning in terms of its content: "Mathematics learning is learning computational methods" (Hn-S6-2-M). Another student looked at mathematics learning in terms of its applications. "Mathematics learning is for applications in daily life" (Cn-S6-3-L). Yet another student looked at mathematics learning in terms of academic achievement: "Mathematics learning is very special. You can get a pass even without doing much revision. But if you want to get good results, there is a certain degree of difficulty" (Hn-S6-1-M).

Only one student perceived mathematics learning as interesting.

Conception of mathematics classroom

Primary Three

Primary 3 students perceived the mathematics class mainly as a time for learning mathematics. It is interesting to find that homework was a prominent feature in the answers of two students.

> The mathematics class is about a lot of difficult homework. (Tg-P3-1-M)
>
> The mathematics class is about very little homework. (Tg-P3-2-M)

Two students perceived the mathematics class as "very interesting".

Primary Six

The theme of learning dominated the answers of Primary 6 students. They saw the mathematics class as a time for "learning", "knowing", "understanding", and "revising" mathematics. There was one student who looked at the mathematics class from the perspective of teaching, rather than learning: "The mathematics class takes place when the teacher teaches us mathematics" (Tg-P6-2-M).

Students expressed mixed feelings, both positive (i.e., “interesting”, “meaningful”) and negative (i.e. “relatively boring”), towards the mathematics class. However, students, in general, felt positively about the mathematics class. One student saw the mathematics class as an opportunity to approach their teachers when they had queries: “The mathematics class is a place where we can ask our teacher if we cannot understand some of the content” (Md-P6-2-H).

Secondary Three

Learning was a dominant theme in the responses of Secondary 3 students. They perceived the mathematics class as a time for learning mathematics. Secondary 3 students expressed both positive and negative feelings towards the mathematics class, but it seems that negative feelings (i.e., “very plain”, “boring”, “relatively difficult”) outweighed positive feelings (i.e., “quite lively”, “quite interesting”, “interesting”).

Some students looked at the mathematics class in terms of their functions (e.g., consulting teachers about difficult problems) and content (e.g., learning different mathematical concepts and computational methods).

Secondary Four

The theme of learning dominated the responses of Secondary 4 students. In general, Secondary 4 students perceived the mathematics class as a time for learning, knowing and applying mathematics. Another salient theme was that Secondary 4 students expressed more positive than negative feelings towards the mathematics class. They found the mathematics class “lively and interesting”, “filled with imagination”, and “interesting, happy and challenging”. The negative feelings included “very boring” and “most hated”. One student expressed other emotions towards the mathematics class: “In the mathematics class, I have two very extreme feelings - achievement and failure”. (Sm-S4-3-L)

In addition, students perceived the mathematics class in terms of their functions and content and the applications of what was learned in those lessons. We got:

> The mathematics class is to help us pass examinations and to teach us to apply mathematics to daily life. (SP-S4-2-H)
>
> In the mathematics class, we learn different computational methods. (SP-S4-3-H)
>
> The mathematics class teaches us to apply mathematics in daily life. (SP-S4-1-H)

Secondary Six

The theme of learning also dominated the responses of Secondary 6 students. They generally perceived the mathematics class as a time for learning and understanding mathematics. On the other hand, one student saw the mathematics class from the perspective of teaching: "In the mathematics class, the teacher instructs the students on computation". (Hn-S6-2-M)

Most Secondary 6 students perceived the mathematics class in terms of their content - what was learned or taught. These responses included "learning basic principles in mathematics", "learning computation methods", and "learning the teacher's ways of thinking". Students expressed both positive and negative feelings towards the mathematics class, but positive responses (i.e., "happy", "lively", "not boring") outnumbered negative ones (i.e., "can be boring"). One student expressed other feelings towards the mathematics class: "In the mathematics class, students experience highs and lows according to how well they cope with the subject" (Ying-S6-2-H).

Summary

In sum, Primary 3 students generally perceived mathematics as something interesting. They perceived the mathematics class as a time for learning mathematics and saw homework as an important component of mathematics learning. The theme of mathematics being a difficult subject began to emerge at Primary 6. In fact, negative feeling about the mathematics classroom intensified at this grade level. As the student moved up the grade levels, mathematics was found to be more and more abstract and academically oriented. In general, mathematics was perceived as a subject of "calculables", useful to

daily life and involved thinking. The theme was more marked at higher levels such as Secondary 4 where the students made explicit expositions such as "mathematics makes use of quantitative data and formulae to work out unknown solutions through computation", "mathematics is a subject that translates abstract principles into concrete computation", and "mathematics is a tool for logical thinking and a tool for data organisation and analysis". This was consistent with what was found in previous studies.

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