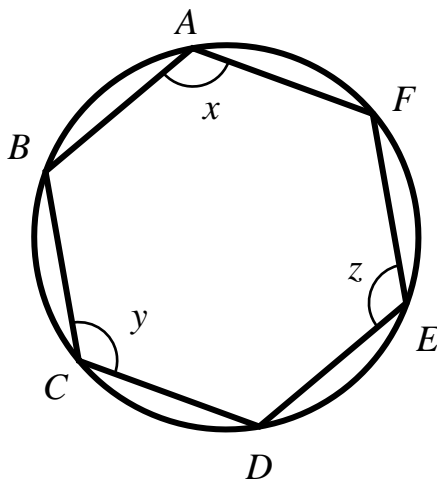# A geometry problem with different solutions

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In a Form 4 classroom, the teacher gave to his students the following question taken from the textbook (Chan, Leung & Kwok, 1997a, p. 204) as class exercise.
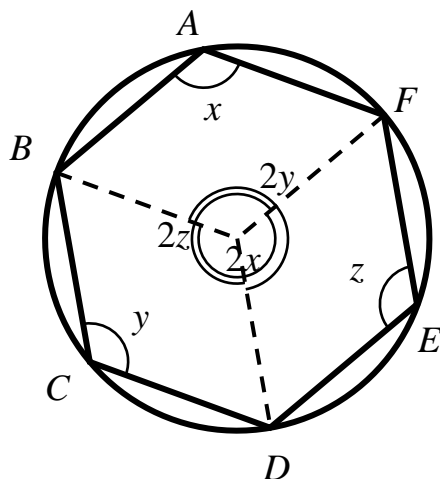
Question:

In the figure, A , B , C , D , E and F are points on the circumference of a circle. Find the value of $x + y + z$.

The following is a suggested solution in the teacher's reference (Chan, Leung & Kwok, 1997b).
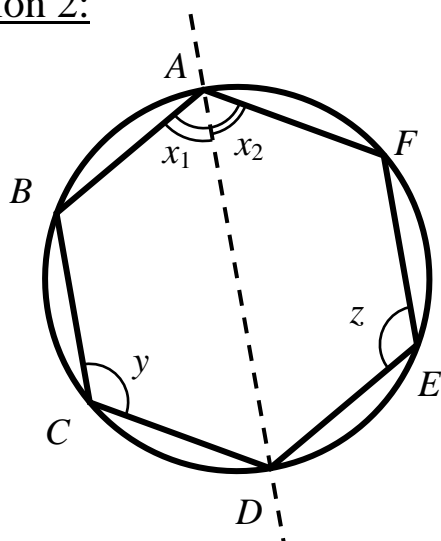
Solution 1:

$2x + 2y + 2z = 2 \times 360°$

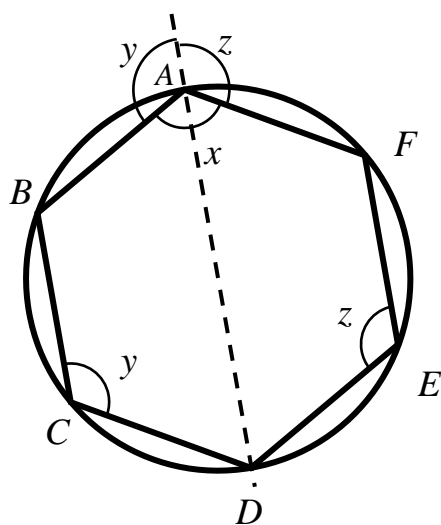$\angle$ at centre $= 2 \angle$ at circumference

To the surprise of the teacher, his students offered many different methods of solving this question.   The following are three different methods given by students.
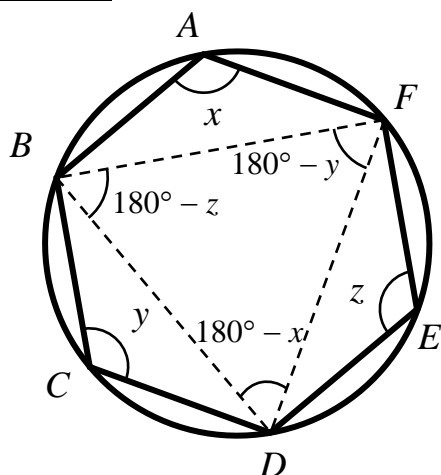
Solution 2:

$x_1 + y = x_2 + z = 180°$
opposite $\angle$s, cyclic quadrilateral

Solution 3:

exterior $\angle$ of cyclic quadrilateral

Solution 4:

opposite $\angle$s, cyclic quadrilateral

Comments on these four solutions

The ideas of Solution 2 and Solution 3 are similar. They are simple. Solution 4 is rather creative and demonstrates the use of symmetric properties in this question. The solution given in the teacher's reference, Solution 1, though rather tedious, does not need to use knowledge beyond the foundation part. It can serve as an enrichment exercise for students who study the foundation part only.

Reflection about teaching

What the author wishes to point out is: don't under-estimate the ability of students! If enough freedom and opportunities are given to students (and perhaps together with suitable guidance from the teacher), they can come up with unexpected, sensible performance. We sometimes complain that our students are reluctant to think. It is perhaps because we do not give them a chance!

Remark

A common mistake for this question is that students assume that the hexagon is regular and shout out the answer immediately.

Acknowledgement

References

Chan, M.H., Leung, S.W. and Kwok, P.M. (1997a), *Pleasurable Learning Mathematics (Book 4A)*.Hong Kong: Chung Tai.

Chan, M.H., Leung, S.W. and Kwok, P.M. (1997b), *Pleasurable Learning Mathematics -- Teacher's Reference (Book 4A)*.Hong Kong: Chung Tai.