一個數學世界？
抑或多個數學世界？
—— STEM中的M
是個怎樣的數學世界？

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2019.06.21

楔子

個人經歷的兩個小故事

- 其一發生於四十多年前
- 其二發生於兩個月前

成文迄今，倏忽廿載。文內述及的想法與處理手法，於今天而言未必盡同，但總的基本想法仍然一樣

蕭文強
1997.6.6

INT. J. MATH. EDUC. SCI. TECHNOL., 1977, VOL. 8, NO. 1, 17–21

Mathematics for math-haters

by MAN-KEUNG SIU
Department of Mathematics, University of Hong Kong, Hong Kong

(*Received 7 May* 1976)

This article attem... answer the following ...ons about a course commonly known as ' Mathematics for Liberal Arts Students ', based on the author's thought and experience gathered from teaching such a course: (1) Why should there be such a course? (2) What do we want to get across? (3) What can we talk about in the course?

1. Prologue

In American college catalogues one usually finds a course described as ' Mathematics in the Humanities ', or ' Mathematics for Liberal Arts Students ', or ' Mathematics Appreciation ', or ' Introduction to Mathematical Thinking ', or ' Mathematics for Poets ', or . . . , which are all euphemisms for ' Mathematics for those who hate mathematics, but have to take it for a course requirement anyway '. Such a course is usually offered ... exemplified by the numerous texts that appe... course, but best described as ' Mathematical ... of popular topics such as set theory, symb... theory, topology, computers, group; (3) as ... mathematics, usually with a slant towards t... These formats have their separate merits ... course is of sufficient significance (in contrast ... dismisses it as just an ' easy course ' for thos... some serious discussion, in the hope of bettering the course with cooperative effort. It is with such hope that the author ventures to describe what he has done while teaching at the University of Miami. No claim is made to any novelty of idea, but it seems that not much discussion has been carried on so far in this important direction. Indeed, the author based his plan on a very old article by Ore [1].

2. Why should there be such a course?

" I am not going to be a mathematician, nor a scientist, nor an engineer, nor an accountant. Mathematics means nothing to me. I don't need it."

" Why should I care whether there is any odd perfect number or not? The world won't be better off even if we know the answer."

" What's the point of doing math? Don't tell me math helps to put a man on the moon. I know all that, but then what's so important about putting a man on the moon while millions are starving on this earth?"

These are all sensible comments about mathematics. In a sense they reveal a disturbing fact about modern day mathematics and mathematicians. We

A course on "Introduction to Mathematical Ideas" taught in 1974-1975

M. K. Siu, Mathematics for math-haters, *International Journal of Mathematical Education in Science and Technology*, Vol. 8, No.1 (1977), 17-21.

Course Sc 511 (later retitled as MATH 2001)

Development of Mathematical Ideas

From 1976 to 2009 it was offered to mathematics students as an elective.

Course YSCN 0002

Mathematics : A Cultural Heritage

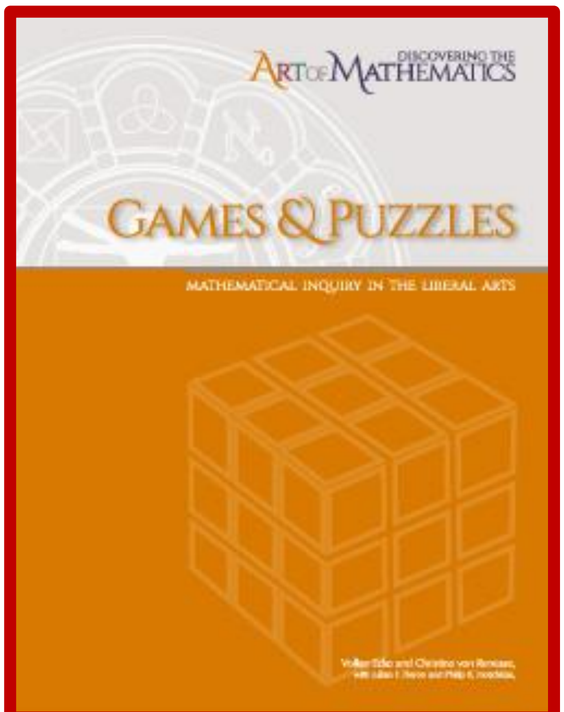
From 1999 to 2009 it was offered as an elective for students in any faculty and department.

One day after class in 1974 a student asked me, “Can you tell me how I can make a lampshade?”

Making a Lampshade

What is the shadow cast by a lampshade on the wall?

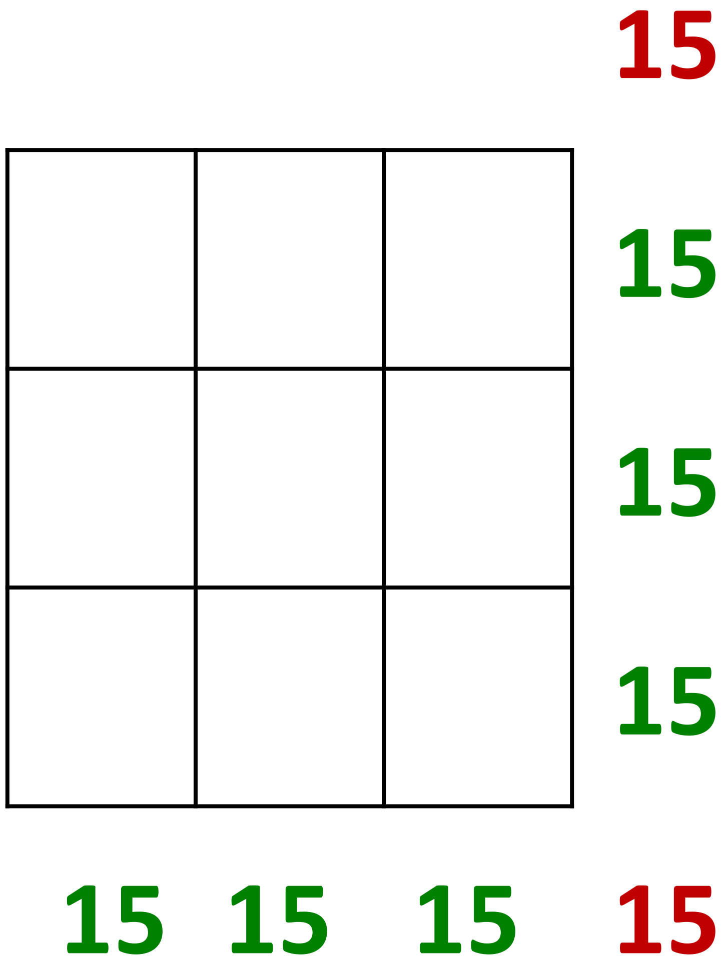
Steven Rossi, Xiao Xiao, Finding a unique solution to Radon-Kaczmarz puzzles, *Pi Mu Epsilon Journal*, 14 (2018), *no*. 9, 573-580.



Games and Puzzles

Volker Ecke and Christine von Renesse, with Julian F. Fleron and Philip K. Hotchkiss, 2015 ; current version 2018 .

Section 3.2 : Radon/Kaczmarz Puzzles



You are given the three row sums, three column sums and the two diagonal sums, fill in the nine cells in the 3 x 3 grid with suitable positive integers chosen from 1 to 9.

			15
2	7	6	15
9	5	1	15
4	3	8	15
15	15	15	15

You are given the three row sums, three column sums and the two diagonal sums, fill in the nine cells in the 3 x 3 grid with suitable positive integers chosen from 1 to 9.

			15
5	5	5	15
5	5	5	15
5	5	5	15
15	15	15	15

You are given the three row sums, three column sums and the two diagonal sums, fill in the nine cells in the 3 x 3 grid with suitable positive integers chosen from 1 to 9.

			15
4	6	5	15
6	5	4	15
5	4	6	15
15	15	15	15

You are given the three row sums, three column sums and the two diagonal sums, fill in the nine cells in the 3 x 3 grid with suitable positive integers chosen from 1 to 9.

			15
3	5	7	15
9	5	1	15
3	5	7	15
15	15	15	15

You are given the three row sums, three column sums and the two diagonal sums, fill in the nine cells in the 3 x 3 grid with suitable positive integers chosen from 1 to 9.

			15
7	2	6	15
4	5	6	15
4	8	3	15
15	15	15	15

You are given the three row sums, three column sums and the two diagonal sums, fill in the nine cells in the 3 x 3 grid with suitable positive integers chosen from 1 to 9.

2	7	6
9	5	1
4	3	8

5	5	5
5	5	5
5	5	5

4	6	5
6	5	4
5	4	6

3	7	5
7	5	3
5	3	7

2	8	5
8	5	2
5	2	8

1	9	5
9	5	1
5	1	9

3	5	7
9	5	1
3	5	7

4	5	6
7	5	3
4	5	6

3	6	6
8	5	2
4	4	7

There are altogether 41 solutions to the puzzle, falling into essentially 9 types with the remaining ones obtained via rotation or reflection.

3	9	3
5	5	5
7	1	7

6	6	3
2	5	8
7	4	4

4	8	3
4	5	6
7	2	6

7	5	3
1	5	9
7	5	3

5	7	3
3	5	7
7	3	5

There are **9** unknowns and **12** equations. Why can't the given conditions pin down the solution?

Even if in addition we are given the five *NW-SE* diagonal sums the answer is still not unique.

3	9	3
5	5	5
7	1	7

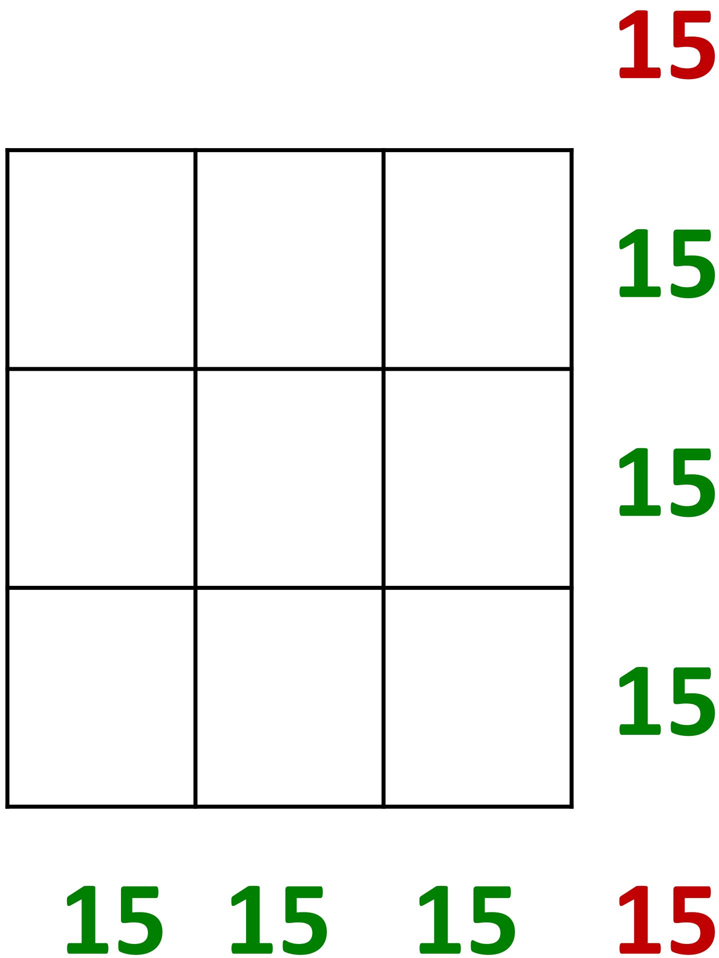
6	6	3
2	5	8
7	4	4

4	8	3
4	5	6
7	2	6

7	5	3
1	5	9
7	5	3

5	7	3
3	5	7
7	3	5

If in addition we are given all the ten diagonal sums, then the answer will be unique.

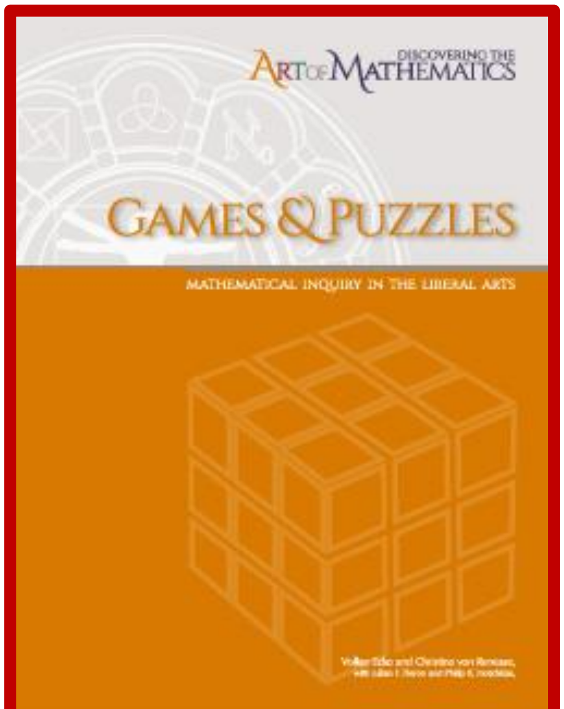


In general we are looking at a system of 8 linear equations with 9 unknowns, the rank of the coefficient matrix being equal to 7. Of the 9 unknowns there are 7 pivotal unknowns and 2 free unknowns.

Question: What sort of conditions will guarantee a unique solution if one exists?

Steven Rossi, Xiao Xiao, Finding a unique solution to Radon-Kaczmarz puzzles, *Pi Mu Epsilon Journal*, 14 (2018), *no*. 9, 573-580.

Created by Julian F. Fleron



Games and Puzzles

Volker Ecke and Christine von Renesse, with Julian F. Fleron and Philip K. Hotchkiss, 2015 ; current version 2018 .

Section 3.2 : Radon/Kaczmarz Puzzles

Kaczmarz's algorithm for solving a system of linear equations, 1937.

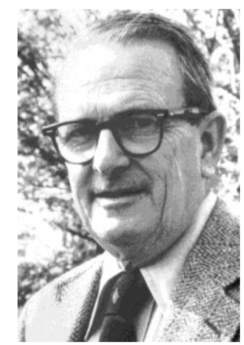
Radon (Inverse) Transform, 1917.

Stefan Kaczmarz (1895-1940)

Johann Radon (1887-1950)

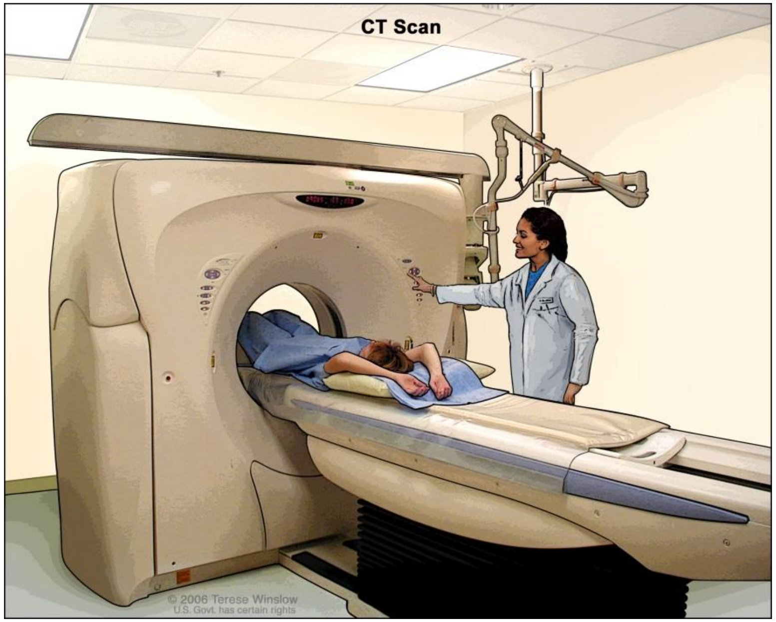
Godfrey Newbold Hounsfield (1919-2004)

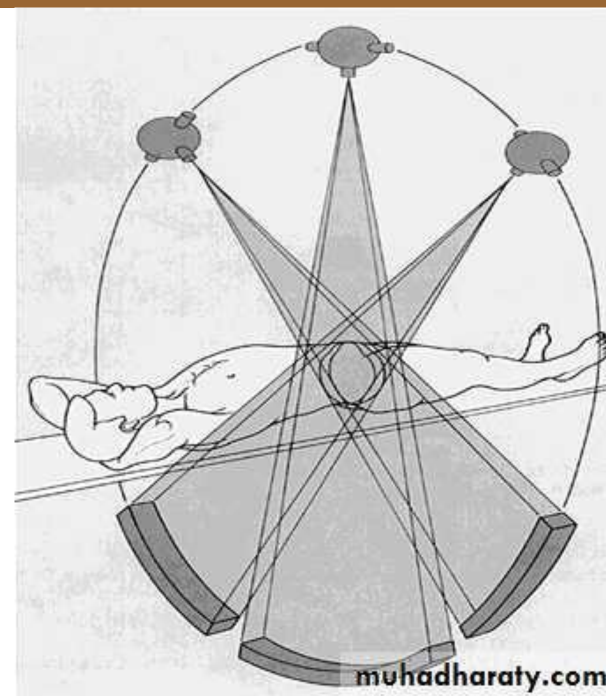
Nobel Prize for Physiology or Medicine for development of diagnostic technique of X-ray CT (computed tomography), 1979.

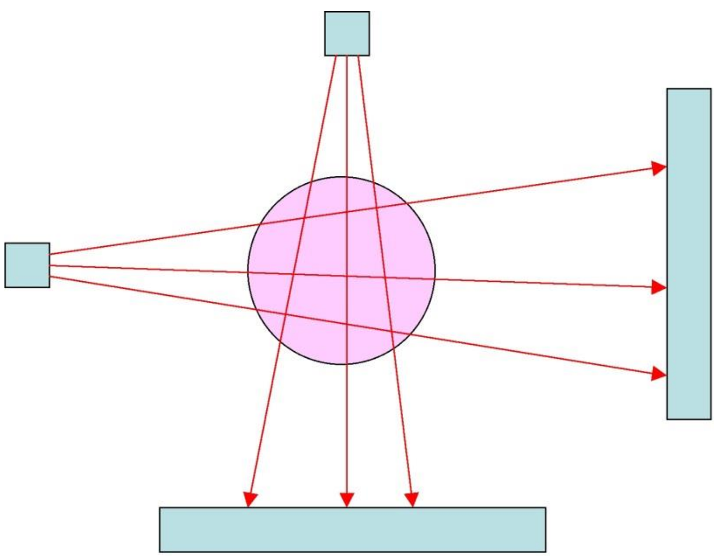


Allan McLeod Cormack (1924-1998)

CT Scan (Computerized Tomography)





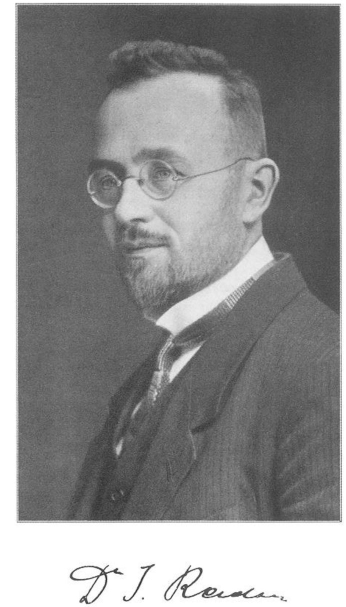


Basically we try to figure out the entries of a large grid knowing the row sums, column sums, diagonal sums, etc.

Kaczmarz's algorithm
for solving a system of
linear equations, 1937.

Mathematics,

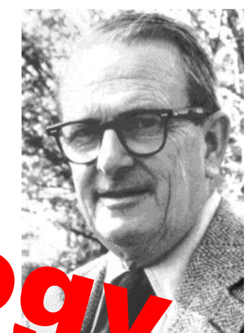
Radon (Inverse) Transform,
1917.



Stefan Kaczmarz
(1895-1940)

Johann Radon
(1887-1950)

Science, Technology, Engineering,



Nobel Prize for Physiology or
Medicine for development of
diagnostic technique of X-ray
CT (computed tomography), 1979.

Godfrey Newbold
Hounsfield
(1919-2004)

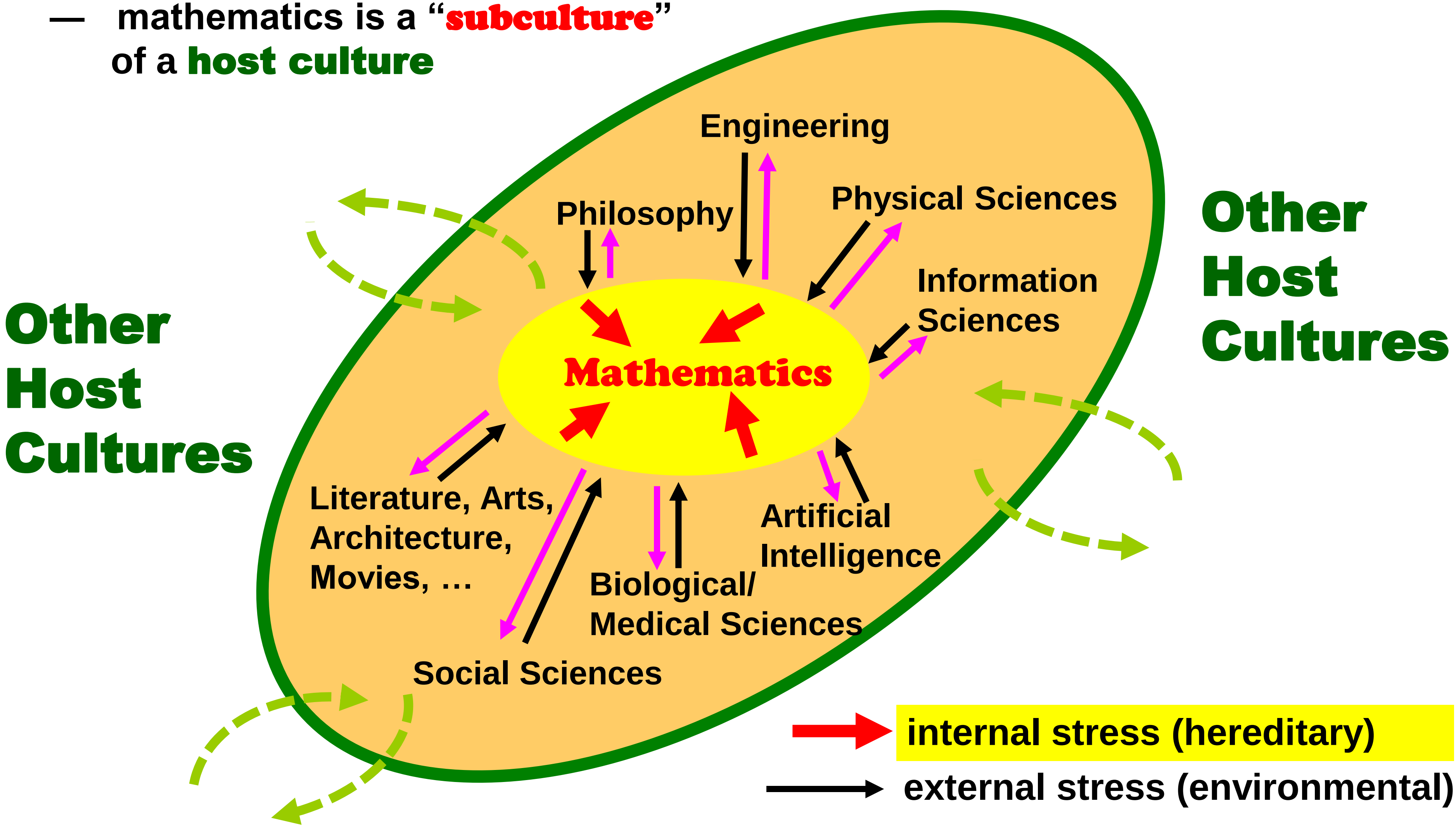
Allan McLeod
Cormack
(1924-1998)

在上個世紀九十年代美國國家科學基金會已經提到要重視科學教育，並且推出SMET (在2001年改為STEM) 這個口號，但似乎在近十年間這個口號才突然受到重視而盛行，更備受標榜，有些人甚至把它當作萬應靈丹！

固然，這種熱潮出現的因素不全是與學理有關，但從學理角度而言，我們作為數學工作者和數學教師，不妨嘗試探討STEM中的 M 是一個怎樣的數學世界。推廣一些，不妨再問下去：只有一個數學世界嗎？抑或有多個數學世界呢？從不同視角多面審察這回事，對學習數學和教授數學應有裨益。

An **evolutionary model** based on view and idea of R.L. Wilder

— mathematics is a “**subculture**” of a **host culture**



internal stress (hereditary)

external stress (environmental)

influence on host culture

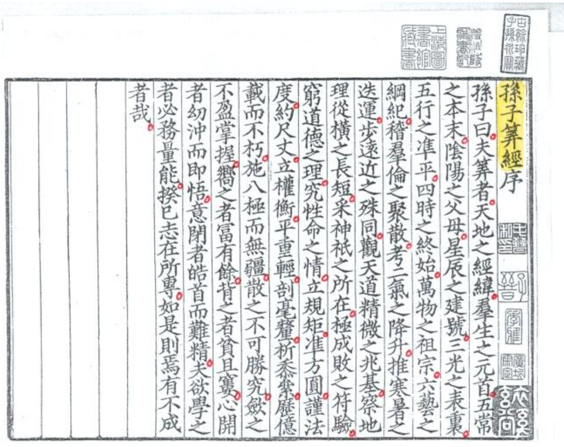
diffusion to and from other host cultures

Raymond Wilder, *Mathematics As a Cultural System*, 1981.

Curiosity, Imagination
(好奇心、想像力)

Disciplined and Critical Thinking
(慎思明辨的頭腦)

[precision in **mathematics**
as well as in **words**
以精確**數學**推導論證，
以清晰**語言**表達溝通。]

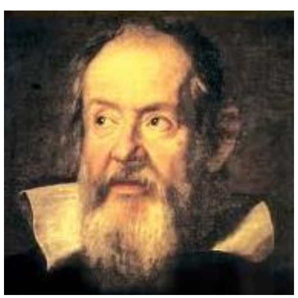

孫子筭經序
孫子曰夫筭者天地之經緯羣生之元首五常
之本末隂陽之父母星辰之建號三光之表裏
五行之準平四時之終始萬物之祖宗六藝之
綱紀稽羣倫之聚散考二氣之降升推寒暑之
迭運步遠近之殊同觀天道精微之兆基察地
理從橫之長短采神祇之所在極成敗之符驗
窮道德之理究性命之情立規矩準方圓謹法
度約尺丈立權衡平重輕剖毫釐析黍絫歷億
載而不朽施八極而無疆散之不可勝究斂之
不盈掌握嚮之者富有餘背之者貧且窶心開
者幼沖而即悟意閉者皓首而難精夫欲學之
者必務量能揆已志在所專如是則焉有不成
者哉

《孫子算經》
(公元四/五世紀)

孫子曰：夫算者，天地之經緯，群生之元首；五常之本末，陰陽之父母；星辰之建號，三光之表裏；五行之準平，四時之終始；萬物之祖宗，六藝之綱紀。

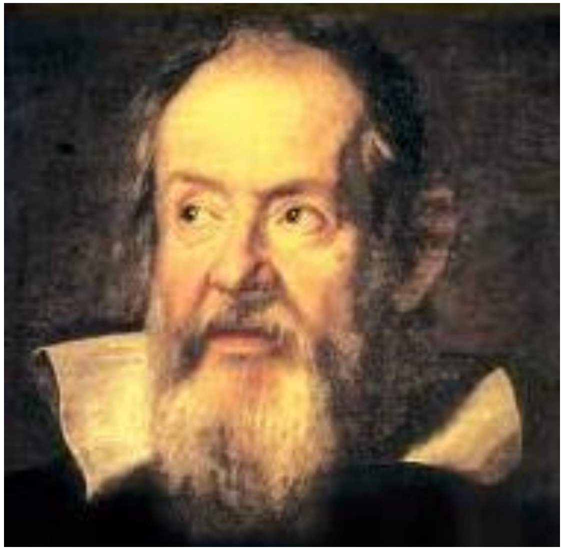
稽群倫之聚散，考二氣之降升；推寒暑之迭運，步遠近之殊同；觀天道精微之兆基，察地理從橫之長短；采神祇之所在，極成敗之符驗；窮道德之理，究性命之情。立規矩，準方圓，謹法度，約尺丈，立權衡，平重輕，剖毫釐，析黍絫；歷億載而不朽，施八極而無疆。散之不可勝究，斂之不盈掌握。嚮之者富有餘，背之者貧且窶；心開者幼沖而即悟，意閉者皓首而難精。夫欲學之者必務量能揆己，志在所專。如是則焉有不成者哉。

「大自然的奧秘，都寫在這本永遠展開在我們面前的偉大書本裏。如果我們不先學曉書本所用的語言，就不能理解它。**這本書，是用數學語言寫成的。**」



Galileo Galilei
(1564-1642)

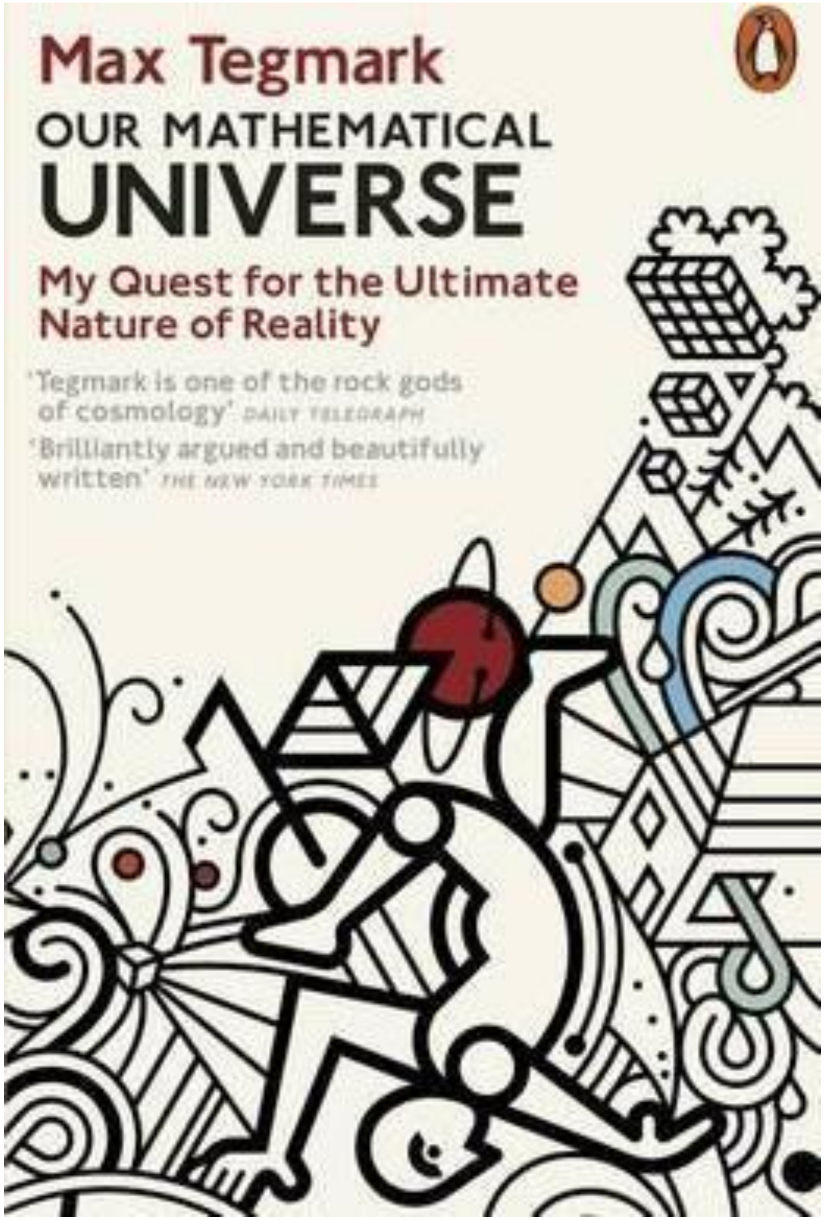
Il Saggiatore (The Assayer)
Letter to the Illustrious and Very Reverend Don Virginio Cesarini
from Galileo Galilei (1623)



Galileo Galilei
(1564-1642)

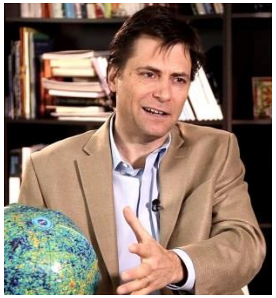
HOW (MUCH) rather than WHY?

[a quantitative rather than a qualitative description]



"But why has our physical world revealed such extreme mathematical regularity [...] we'll explore [...] a crazy-sounding belief of mine that our physical world not only is *described* by mathematics, but that it *is* mathematics, making us self-aware parts of a giant mathematical object."

Chapter 1: What Is Reality? (p. 6)



Max Tegmark, *Our Mathematical Universe: My Quest for the Ultimate Nature of Reality* (2014).

Max Erik Tegmark (1967-)

You are living in a world of mathematics!

INT. J. MATH. EDUC. SCI. TECHNOL., 1984, VOL. 15, NO. 1, 47–52

You are living in a world of mathematics

by MAN-KEUNG SIU and NAM-KIU TSING

Department of Mathematics, University of Hong Kong, Hong Kong

(*Received 24 September 1982*)

• • • • • • • • • •

All of us will probably agree that mathematics is essential to the advancement of science and technology; it finds useful applications in various disciplines such as physics, chemistry, biology, engineering, economics, management, etc. However, ironic as it may seem, most of the time people are so dazzled by its achievements that they forget about mathematics itself. Allan L. Hammond, of the magazine *Science*, refers to mathematics as 'our invisible culture'. Paul R. Halmos, a noted mathema-

Why is it like that?

Probably the long history of mathematics has something to do with it. Even before other sciences began their modern forms, mathematics had already some two thousand years of illustrious accomplishment behind it; only a small portion of this is learnt in school. Even at the university level, while students in other sciences proceed from post-19th-century development to the latest achievement in the 20th century, their knowledge of mathematics stops (more or less) at the beginning of the 19th century! Thus mathematics gradually acquires a language of its own, which sounds foreign and inaccessible to people not in the field. Besides, mathematics is a subject with a cumulative nature; its past is forever assimilated in its present and future. By its nature, mathematics involves abstract thinking, and one must put in the requisite amount of effort and time in order to come to grips with it. Although it may be too much to ask us all to put in that amount of effort and time, it is possible and desirable to let each of us become aware of this human endeavour called mathematics, along with its social impact and its relevance to other human activities.

• • • • • • • • • •

1980.11.08-09 (HKU Open Days)

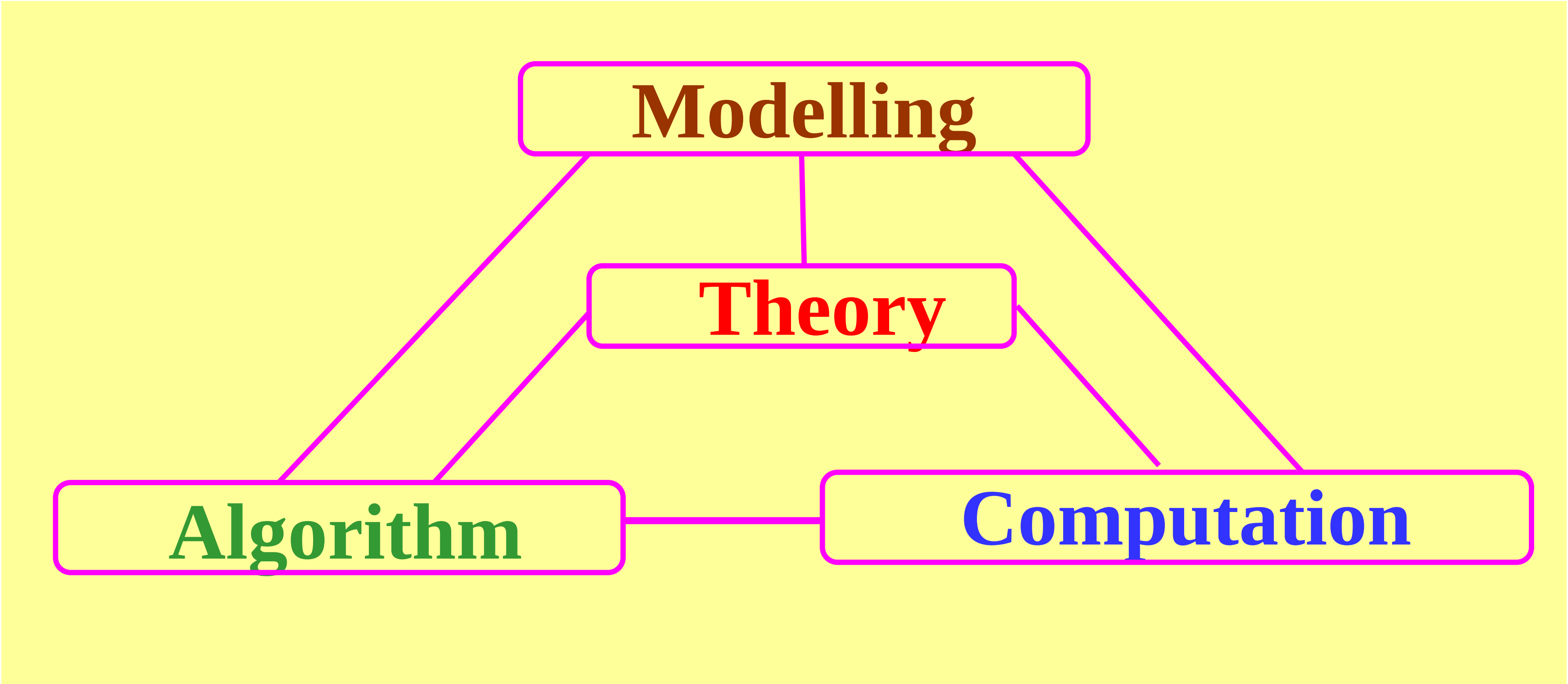
M. K. Siu, N. K. Tsing, You are living in a world of mathematics, *International Journal of Mathematics Education in Science and Technology*, 15 (1), (1984), 47-52.

About a slide show made in 1980

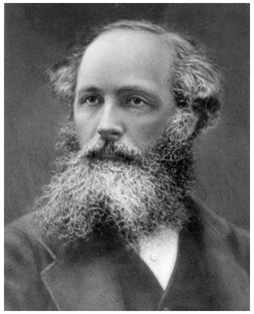
Department of Mathematics of HKU

Received in 1981 a CASME (Commonwealth Association of Science & Mathematics Educators) Award, jointly with N.K. Tsing.

Mathematics



- M. K. Siu, "Algorithmic Mathematics" and "Dialectic Mathematics": The "*Yin*" and "*Yang*" in mathematics education, Invited Lecture at the Second International Conference on the Teaching of Mathematics at the Undergraduate Level, Crete, July 2002.
- M. K. Siu, The algorithmic and dialectic aspects in proof and proving, Presentation at the 19th ICMI Study 19 on Proof and Proving, Taipei, May, 2009.



James Clerk Maxwell
(1831-1879)

$$\nabla \cdot \mathbf{D} = \rho$$
$$\nabla \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$
$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

1865

Maxwell's Equations

James Clerk Maxwell
(1831-1879)

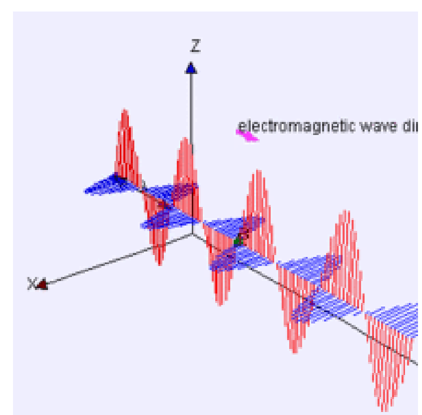
- **J. C. Maxwell, A dynamical theory of the electromagnetic field, *Philosophical Transactions of the Royal Society of London*, 155, 1865, 459-512.**

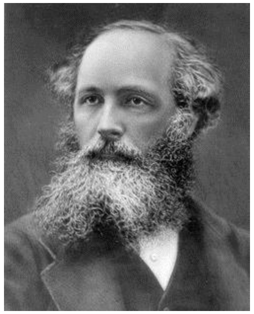
- **J. C. Maxwell, On a method of making a direct comparison of electrostatic with electromagnetic force; with a note on the electromagnetic theory of light, *Philosophical Transactions of the Royal Society of London*, 158, 1868, 643-657.**

- **J. C. Maxwell, *A Treatise on Electricity and Magnetism*, Oxford University Press, 1873.**

Mathematics
(differential equations)

Science
(waves)





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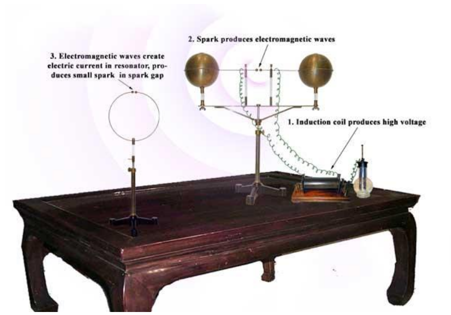
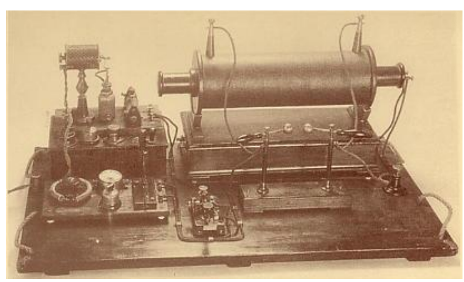
1865

Heinrich Rudolf Hertz
(1875-1894)



1886

Guglielmo Marconi
(1874-1937)



1896

S1

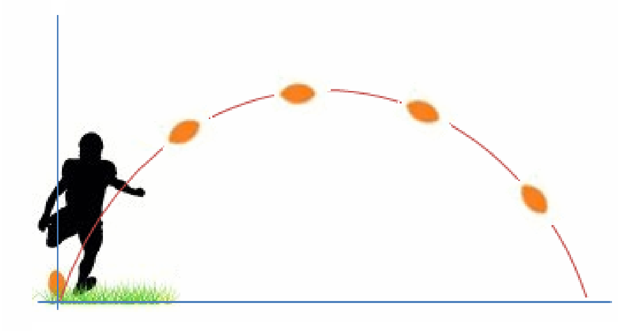
Basic Science
「自然科學」

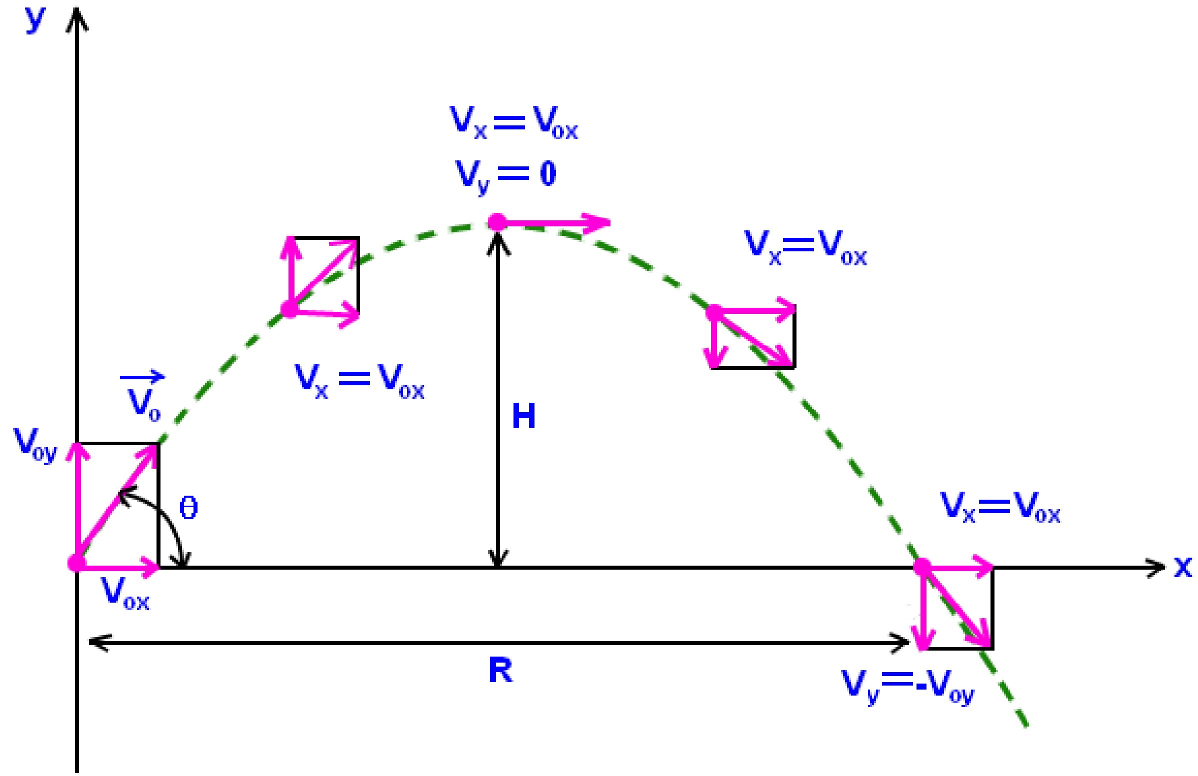
S2

**Research
& Development**
「技術科學」

T

Technology
「技術」





Projectile 拋體運動

http://ggbtu.be/m1082291

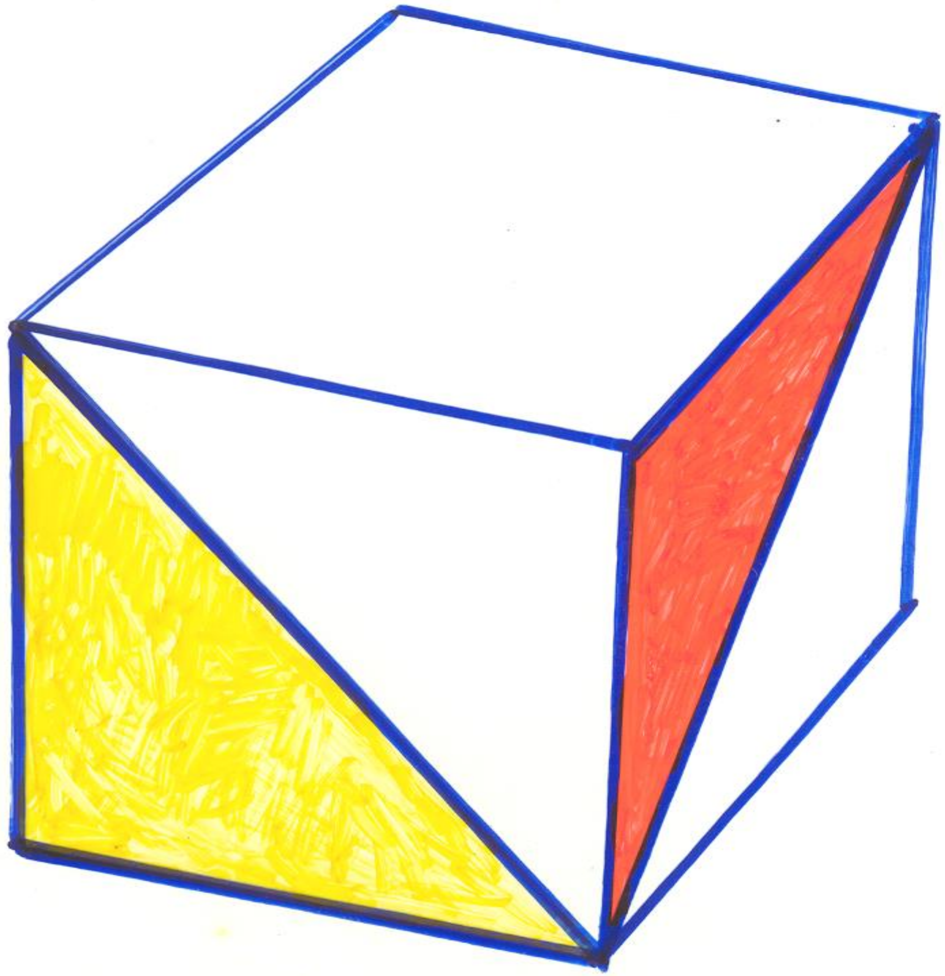
寓數於形，表形以數。
數形結合，雙翼齊飛。

「算術及幾何，
天文學家藉著
這對翅膀翱翔
天際，與天比
高。」

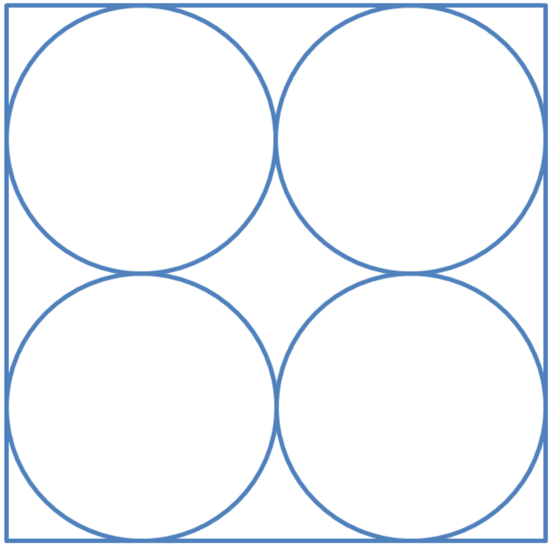
Robert Boyle
(1627-1691)

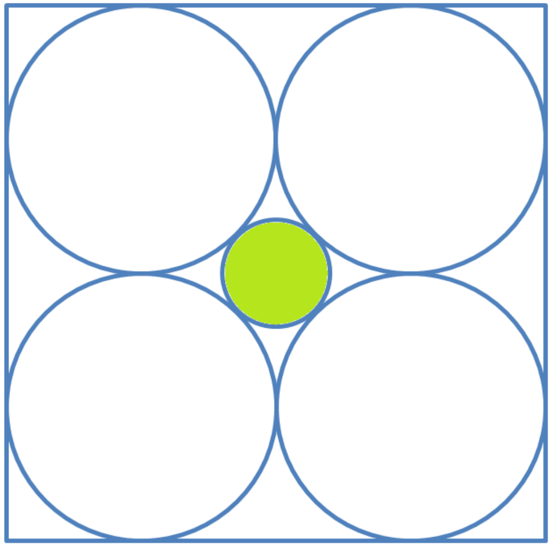
從小學到中學，
大家一定作過不少
計算，對算術世界
不會陌生。

你對幾何世界
有多熟悉呢？

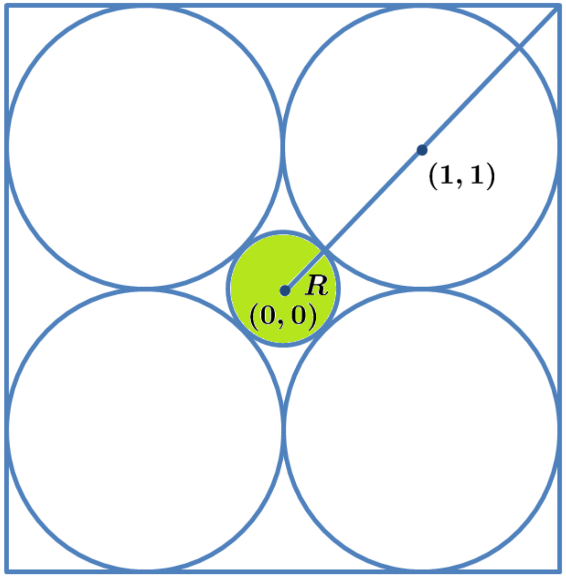


N. Rouche, Reaction to papers on geometry, in *One Hundred Years of L'Enseignement Mathématique: Moments of Mathematics Education in the Twentieth Century*, ed. D. Coray et al, 2003, p.156.

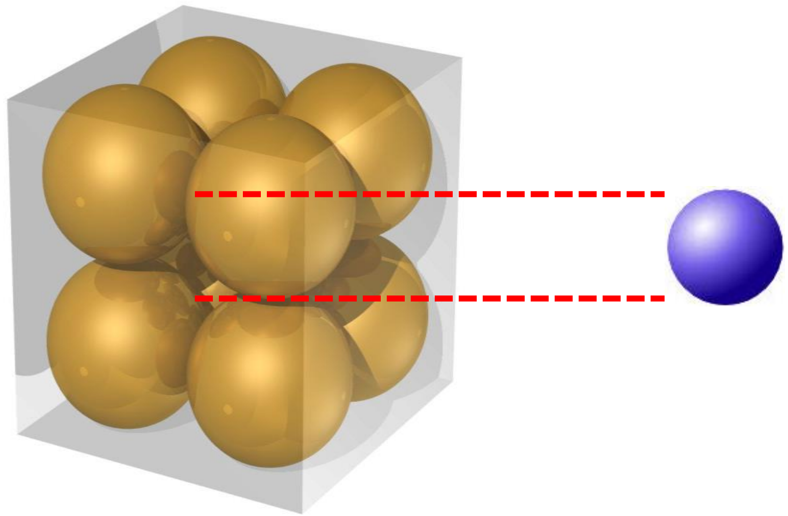




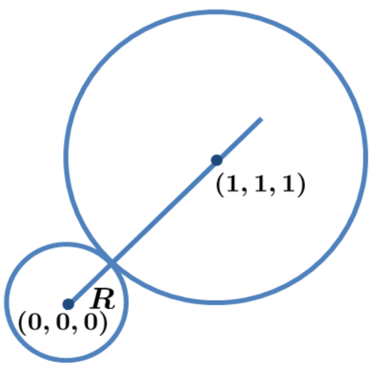
Q. What is the radius of the circle touching all the four circles?



$$R = \sqrt{1^2 + 1^2} - 1$$
$$= \sqrt{2} - 1.$$

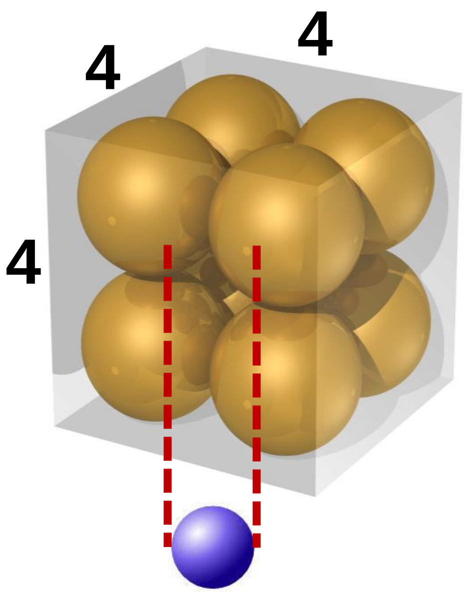


Q. What is the radius of the sphere touching all the eight spheres?



$$R = \sqrt{1^2 + 1^2 + 1^2} - 1 = \sqrt{3} - 1.$$

What happens in the n-dimensional case?

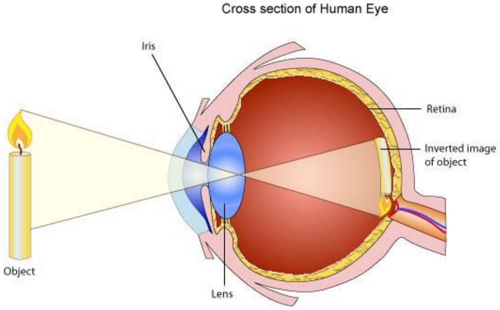


radius of the *n*-ball touching all those *n*-balls situated at the corners of the *n*-cube with a side of length 4 = $\sqrt{n} - 1$

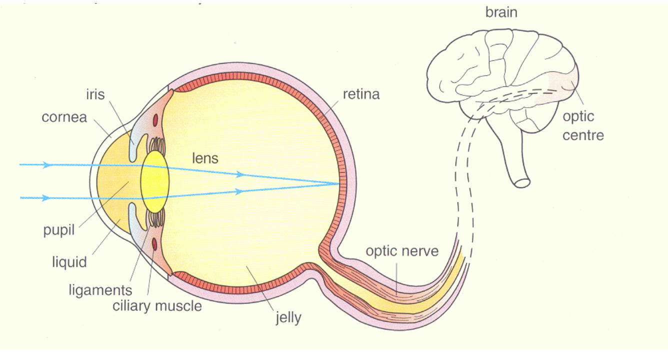
n	$\sqrt{n} - 1$
2	0.4142•••
3	0.7320•••
4	1
5	1.2360•••
6	1.4494•••
7	1.6457•••
8	1.8284•••
9	2
10	2.1622••• > 2

The *n*-ball in the middle of the *n*-cube pokes out of the *n*-cube, when $n > 9$!

One way to visualize a 3-D object is to look at its slices in 2-D to piece up a picture in mind.

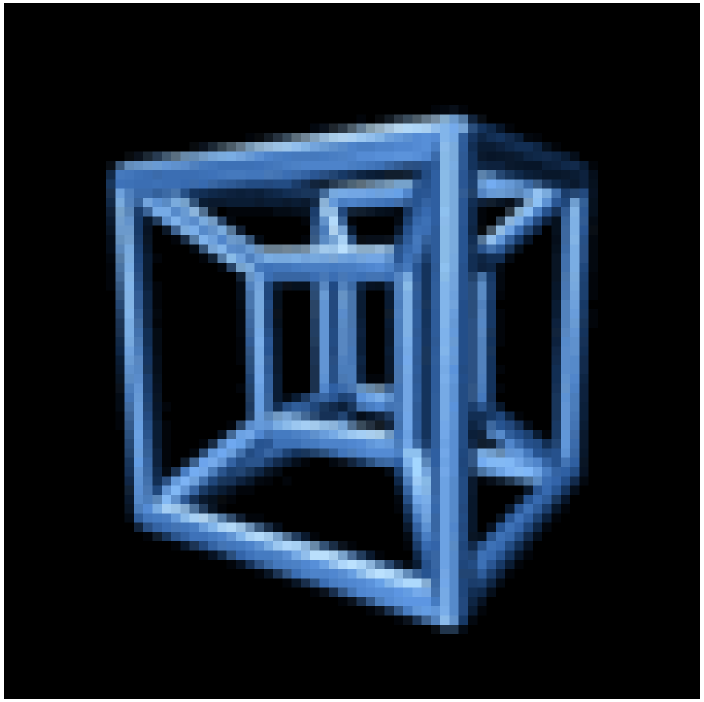


How do we look at this world and interpret what we see?

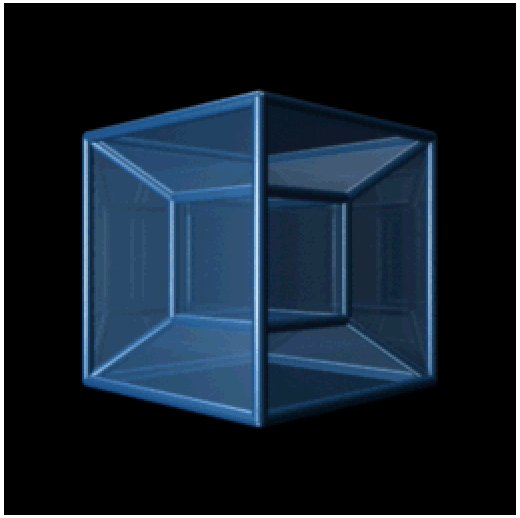


One way to visualize a 3-D object is to look at its slices in 2-D to piece up a picture in mind.

Similarly this can be done for an object in the *N*-D case, *N* > 3 (stepwise down to the 2-D case) . Naturally, it is a much harder process.



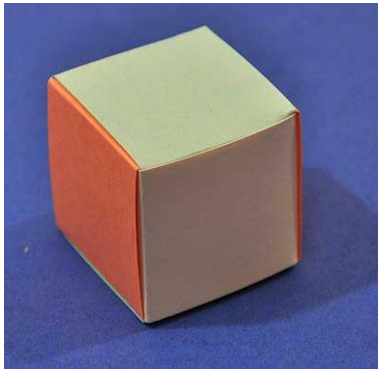
Tesseract



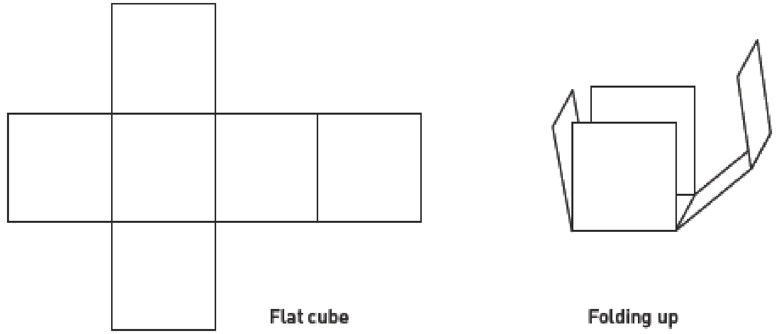
Hypercube

One way to visualize a 3-D object is to look at its slices in 2-D to piece up a picture in mind.

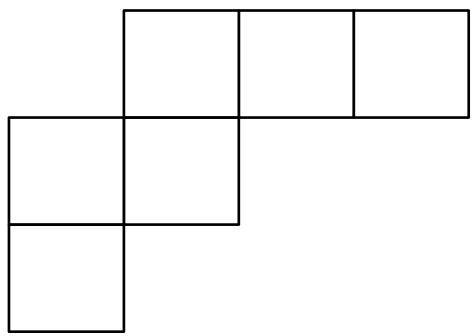
A second way to visualize a 3-D object is to look at its net in 2-D, again, to piece up a picture in the mind.



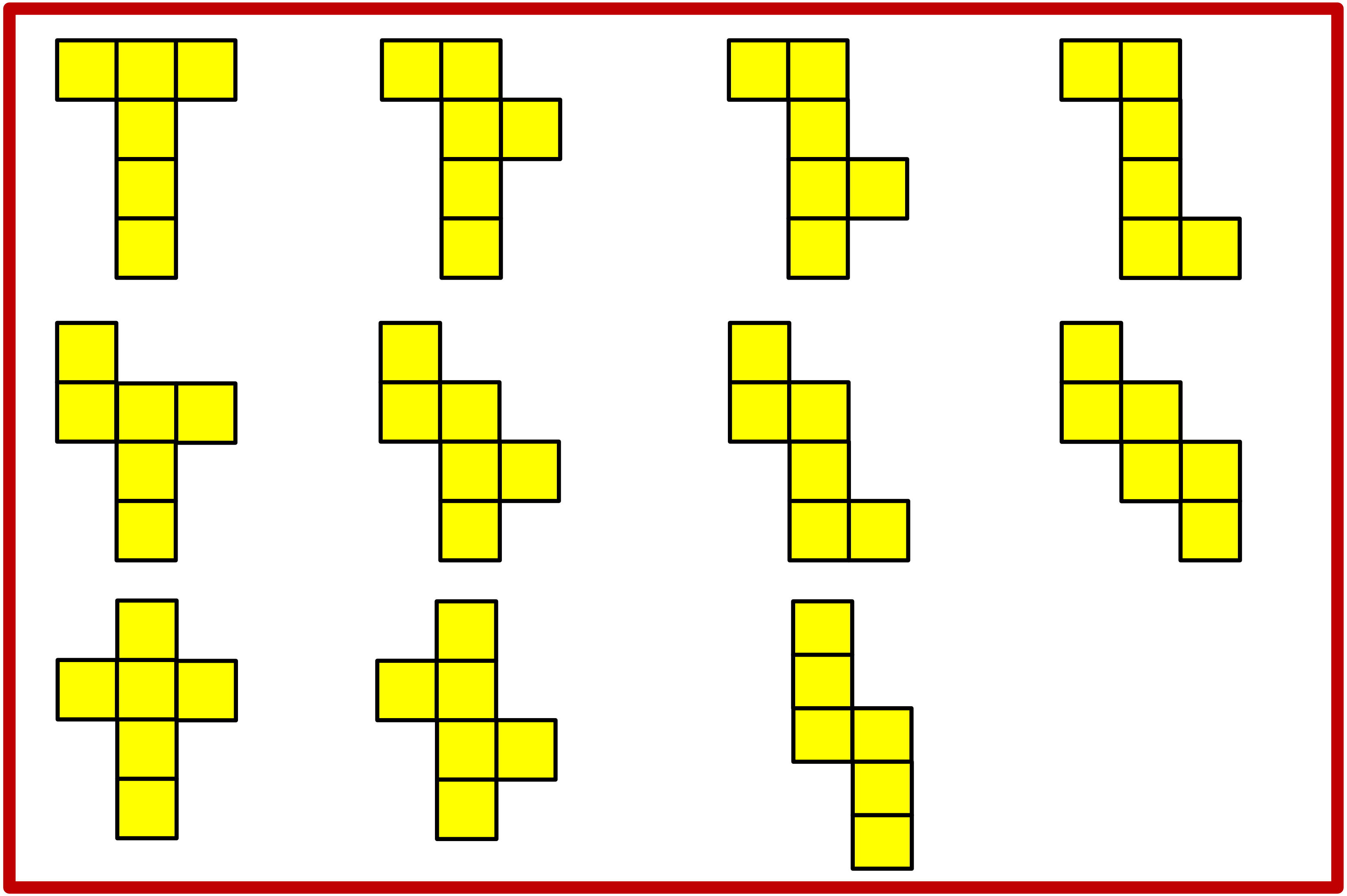
http://ggbtu.be/m185580



Net of a cube



Question: Is this a net of a cube ?



There are altogether only 11 different nets of a cube.

http://ggbtu.be/m184894

A second way to visualize a 3-D object is to look at its net in 2-D, again, to piece up a picture in the mind.

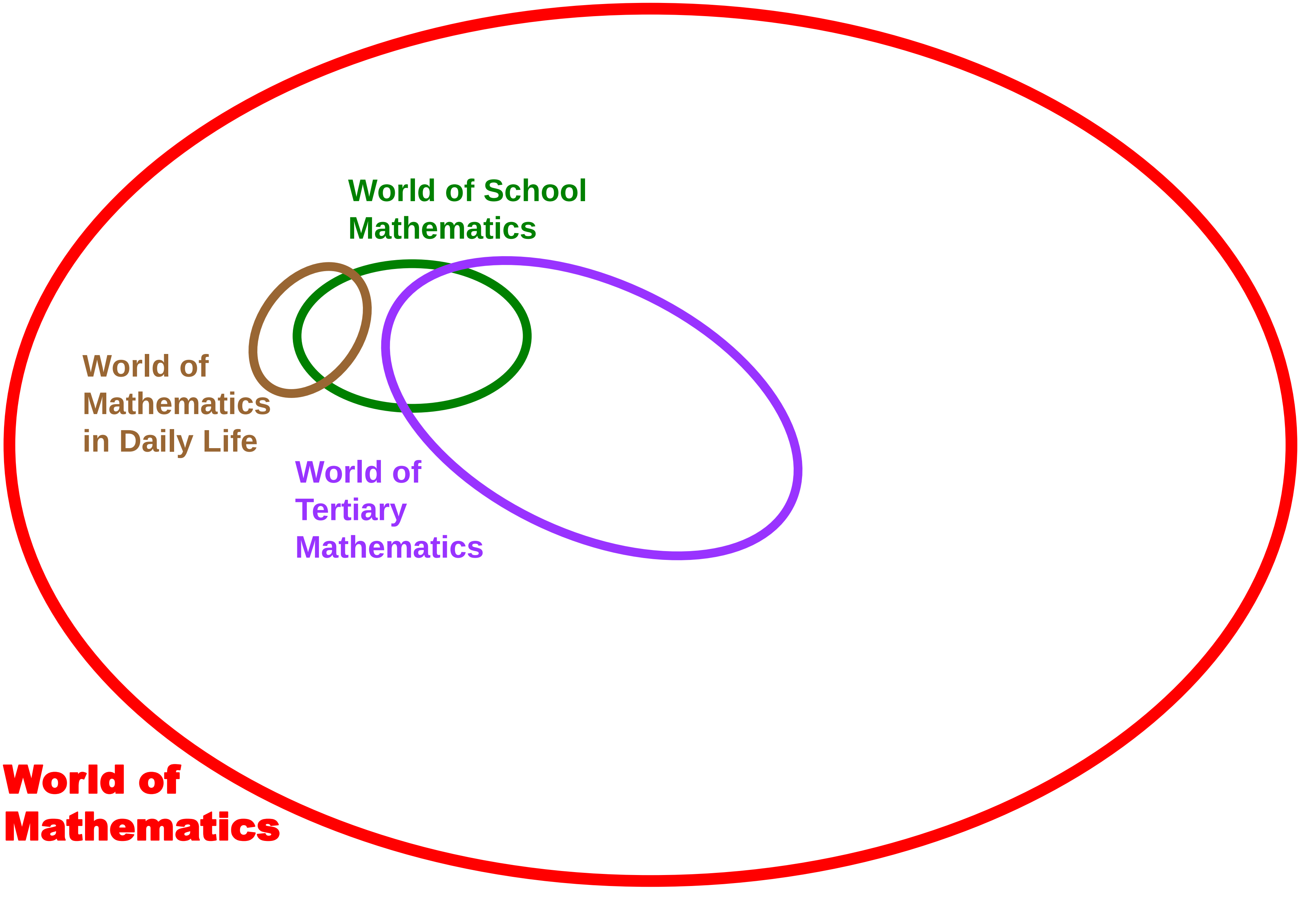
Similarly this can be done for an object in the *N*-D case, $N > 3$ (with the net in an $(N-1)$-dimensional world). Naturally, it is even harder to imagine!

Crucifixion (*Corpus Hypercubus*)

Salvador Dali, 1954

The crucifix is a 3-D net which folds up to form a 4-D hypercube

Salvador Dalí
(1904-1989)



World(s) of Mathematics

- 21st century Skills
- Characteristics of Workplace Mathematics
- Mathematical Competencies that Complement the Work of Computers
- Mathematics in Everyday Life

"Our focus has mainly been on the practical value of mathematics in the world outside school. The goal of mathematics education, however, is also to prepare students for further education to which they add the importance of understanding and appreciating mathematics as a goal in and of itself."

K. Gravemeijer, M. Stephan, C. Julie, F. L. Lin, M. Ohtani, What mathematics education may prepare students for the society of the future? *Int. J. of Sci. And Math. Educ.* 15 (Suppl. 1) (2017), 105-123.

1 € = 8.748 HK$

If I have E €, which amount to H HK$, write down an expression showing the **relationship between E and H**.

Compare!

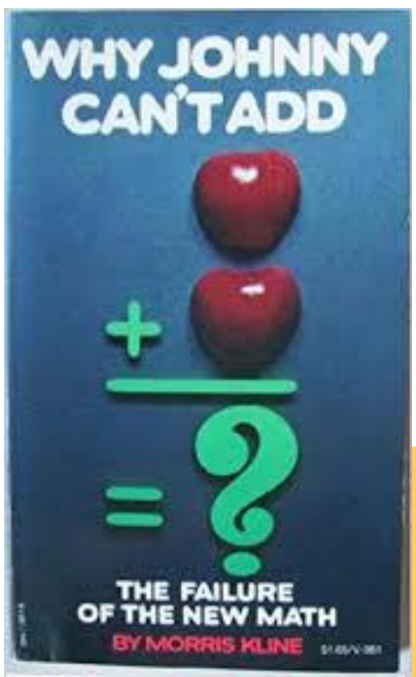
Is it $E = 8.748\,H$,

or $8.748\,E = H$?

The language is different !

Tommy Dreyfus, Why Johnny Can't Prove,
***Educational Studies in Mathematics*, 38 (1999), 85-109.**

"In conclusion, the requirement to explain and justify their reasoning requires students to make the difficult transition from a computational view of mathematics to a view that conceives of mathematics as a field of intimately related structures."

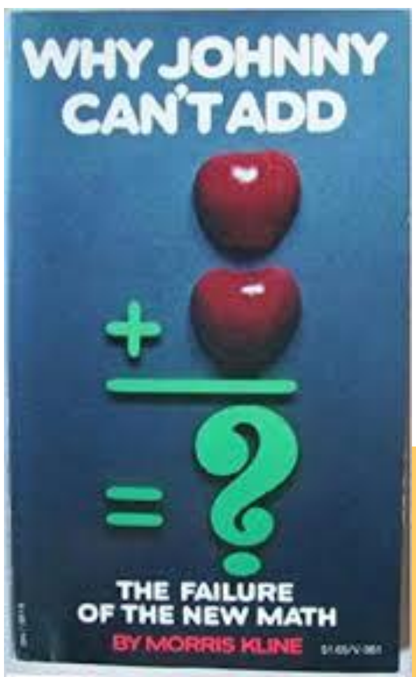


"The deductive proof is the final step. [...] Proofs of whatever nature should be invoked only where the students think they are required. The poof is meaningful when it answers the student's doubts, when it proves what is not obvious."

Morris Kline, *Why Johnny Can't Add: The Failure of the New Math*, Vantage Books, 1974.

不憤不啟，不悱不發

《論語·述而第七》



"The deductive proof is the final step. [...] Proofs of whatever nature should be invoked only where the students think they are required. The poof is meaningful when it answers the student's doubts, when it proves what is not obvious."

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Three types of justification

- **Argument**
- **Proof**
- **Explanation**

R. Duval, Argumenter, démontrer, expliquer; continuité ou rupture cognitive, «*petit x*» 31, 37- 61.

“ Mathematical thought concerning proof is different from thought in all other domains of knowledge, including the sciences as well as everyday experience; *the concept of formal proof is completely outside mainstream thinking.* ”

Education Committee of the EMS, Do theorems admit exceptions? Solid findings in mathematics education on empirical proof schemes, *EMS Newsletter*, 82(2011), 50-53.

What is the main role of a PROOF ?

— "ritual" of the trade?
— for the purpose of verification?
— to guard against error?
— for enhancement of understanding?

蕭文強,《數學證明》, 江蘇教育出版社, 1989; 修訂版, 九章出版社, 2007; 大連理工大學出版社, 2008; 再修訂版, 大連理工大學出版社, 2016.

No right-angle triangle is equilateral.
Some isosceles triangles are equilateral.

Some right-angled triangle is not isosceles.

valid argument ?

No dog is ruminant.
Some quadrupeds are ruminant.

Some dog is not a quadruped.

not valid argument ?

G. Lolli, *QED Fenomenologia della dimonstrazione*, Boringhieri, Torino, 2005, reported in: Education Committee of the EMS, Do theorems admit exceptions? Solid findings in mathematics education on empirical proof schemes, *EMS Newsletter*, 82(2011), 50-53.

No Sright-angle triangle is Mequilateral.
Some Pisosceles triangles are Mequilateral.

Some Sright-angled triangle is not Pisosceles.

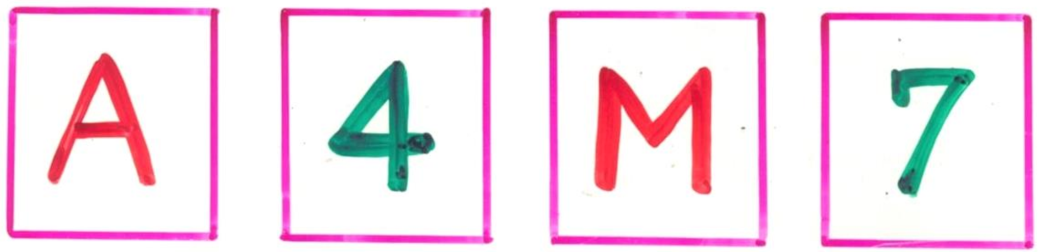
The two arguments are logically identical and not valid !

No Sdog is Mruminant.
Some Pquadrupeds are Mruminant.

Some Sdog is not a Pquadruped.

G. Lolli, *QED Fenomenologia della dimonstrazione*, Boringhieri, Torino, 2005, reported in: Education Committee of the EMS, Do theorems admit exceptions? Solid findings in mathematics education on empirical proof schemes, *EMS Newsletter*, 82(2011), 50-53.

Each card has a number on one side and a letter on the reversed side.



"If a card has A on one side, then it has 4 on the reversed side."

Q. To check the truth of this statement by turning over the *least* number of cards, which cards should you turn over?

P. C. Wason, Reasoning about a rule, Quarterly Journal of Experimental Psychology, 23 (1968), 273-281.

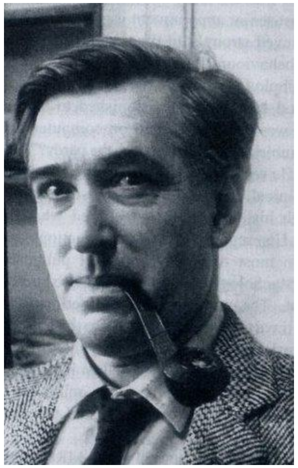
Each card has the age of a person on one side and the beverage that person orders on the reversed side.

Q. To check whether the following rule is violated or not by turning over the *least* number of cards, which cards should you turn over?

No person under 18 can drink beer on the premises.

The accumulated results collected in my classes for a decade since the beginning of 2000 give 37.9% and 88.4% for a correct answer to the first question and the second question respectively.

This classic experiment was devised by the English psychologist Peter C. Wason of University College of London in the mid-1960s in connection with his well-known study of psychology of reasoning.



Peter Cathcart Wason (1924-2003)

"In conclusion, the requirement to explain and justify their reasoning requires students to make the difficult transition from a computational view of mathematics to a view that conceives of mathematics as a field of intimately related structures."

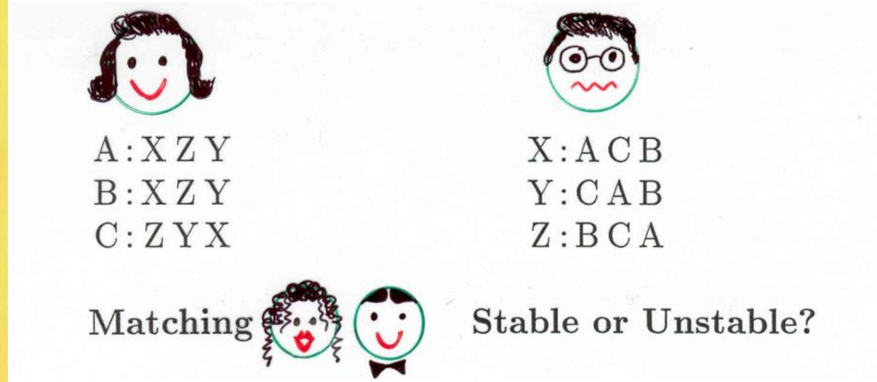
Tommy Dreyfus, Why Johnny Can't Prove, *Educational Studies in Mathematics*, 38 (1999), 85-109.

導致這種對數學的誤解的主要原因，是課堂上的數學往往只著重計算，給學生灌輸各種不同的演算法則和技巧，但求學生能夠快速無誤獲得答案，有了正確答案便是達標了。

至於數學活動當中的探索功夫、提出疑問、從實驗觀察作出歸納、按照邏輯推理作出演繹、提出猜想並試圖證明猜想成立或者找出反例、如此等等，往往忽略了。

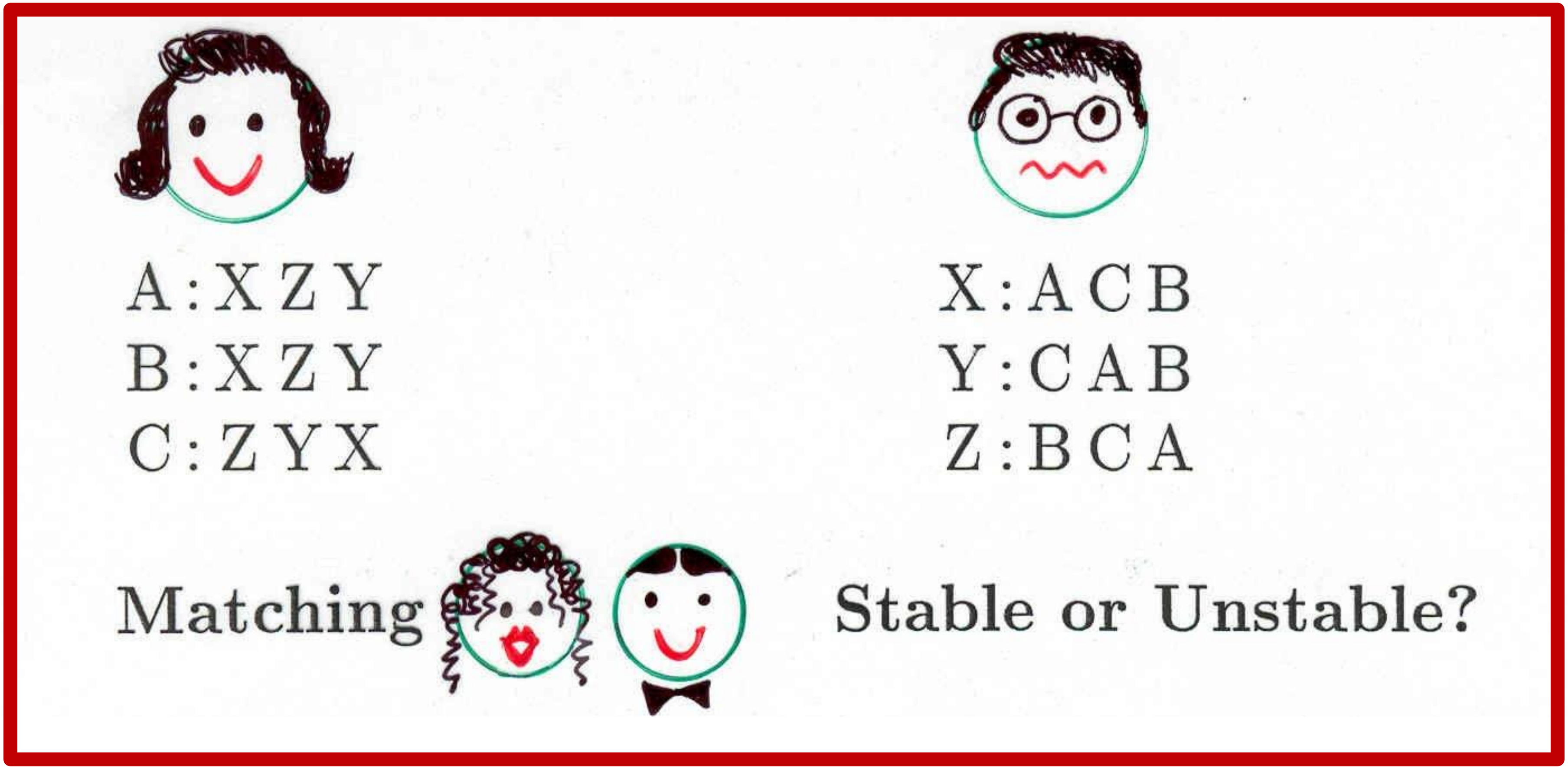
難怪不少學生覺得數學科既枯燥又專制，猶未嘗到數學的真正味道便厭惡或者害怕了這門學科。不少學生也以為數學就是計算，不作任何計算那算是數學呢？

蕭文強，從「數獨」到「讀數」，《數學教育》[*EduMath*], 40(2018), 73-83.



怨偶·推銷員·算死草
—— 淺談組合最優化問題

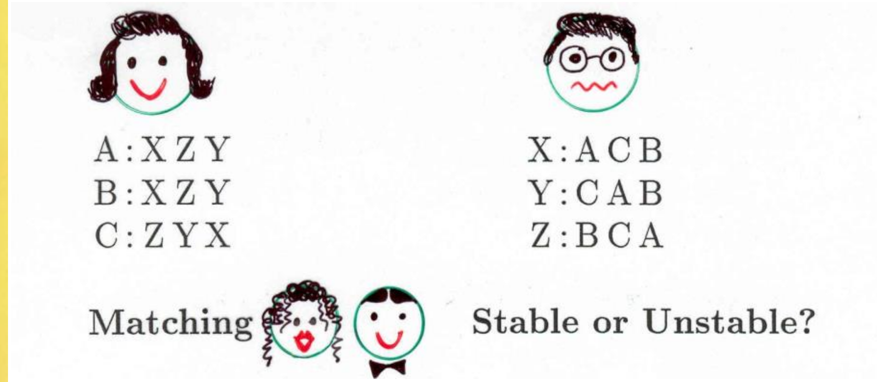
2000年12月2日



(a) AX, BY, CZ ?   unstable

(b) AX, BZ, CY ?   stable

(c) AZ, BY, CX ?   unstable

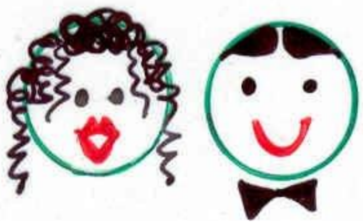


Does a stable matching always exist? How can it be arranged?

怨偶·推銷員·算死草
—— 淺談組合最優化問題
2000年12月2日

A : X Z Y
B : X Z Y
C : Z Y X

X : A C B
Y : C A B
Z : B C A

Matching  Stable or Unstable?

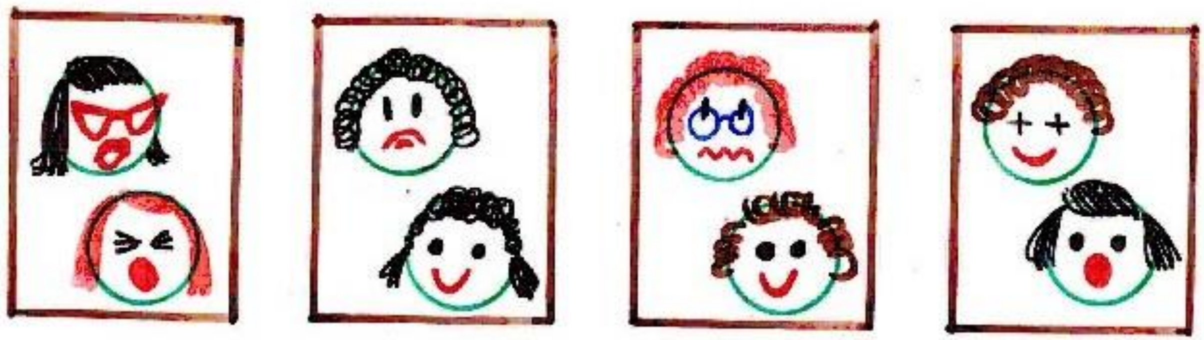
(a) AX, BY, CZ ? unstable

(b) AX, BZ, CY ? stable

(c) AZ, BY, CX ? unstable

A stable matching always exists (by the Gale-Shapley Algorithm).

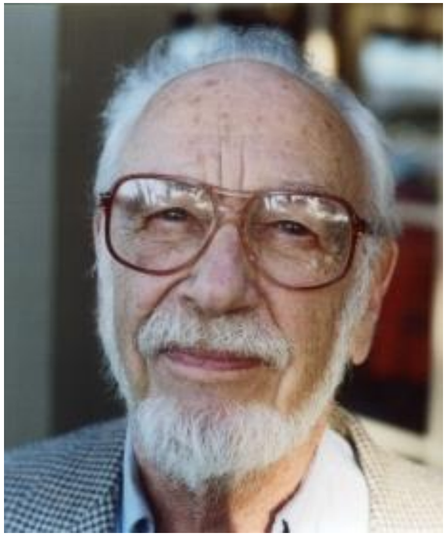
A: D C H G E F B
B: H A C E G D F
etc.



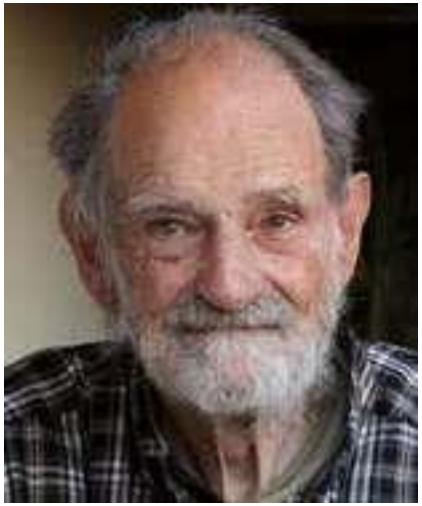
Q. Can the 2N girls be put into N rooms (two to each room) so that there do **not** exist two girls who are not roommates but who prefer each other to their current roommates?

This problem may look similar to the marriage problem, but is essentially different.
A stable matching may be impossible (why?)

D. Gale, L. S. Shapley, College admissions and the stability of marriage, *American Mathematical Monthly*, 96 (1962), 9-15.

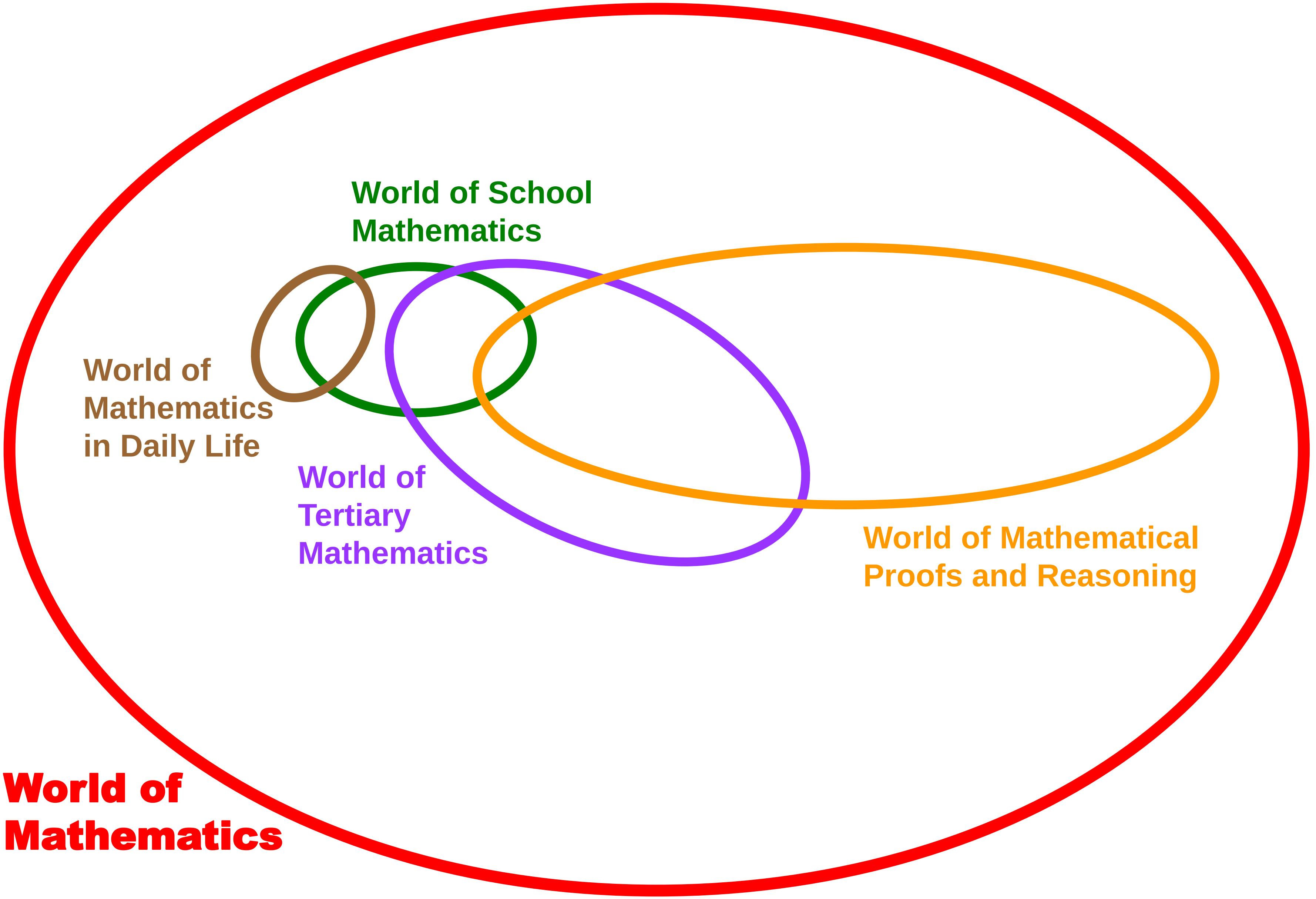


Gale-Shapley Algorithm



David Gale (1921-2008)
John von Neumann Theory Prize 1980 (jointly with Harold W. Kuhn and Albert W. Tucker)

Lloyd Stowell Shapley (1923-2016)
John von Neumann Theory Prize 1981
Nobel Memorial Prize in Economic Sciences 2012



World(s) of Mathematics

明報 (Ming Pao) 2005.05.16

英報形容如「21世紀扭計骰」

The Su-do-ku Game

前港法官掀歐美數字拼圖熱

一種有200年歷史的數字遊戲近來風靡歐美民衆，熱潮之盛直追當年的「扭計骰」。這種日本人稱為「數獨」（Su-doku）的數字拼圖遊戲，要求玩家動腦筋，有人認為它有助於減少患老人癡呆症機會，英國教師最近更提倡用它作為鍛煉學生腦力的活動，英國多份報章更在搶眼位置刊載遊戲供讀者玩耍，藉以刺激銷量。

這種數字拼圖遊戲於18世紀後期由一名瑞士數學家發明，但其後一個多世紀內都未受重視。英國一本雜誌在20餘年前發掘出它的魅力，日本人進一步增加遊戲難度，並把它命名爲「數獨」，意即遊戲由個位數組成。曾任香港高等法院法官的新西蘭人高樂德（Wayne Gould）在去年向英國《泰晤士報》推銷這種遊戲，獲採納刊載，逐漸在英國掀起熱潮。

命名「數獨」意指由個位數組成

「數獨」玩法簡單，遊戲主體是一個由9個九宮格排列成的大正方形，每一行每一列都由9個小方格組成，遊戲編寫者會先在一些小方格裏填上從1到9的不同數字，玩家要填滿其餘空格，規則是大正方形每一行每一列及每個九宮格裏均必須包括1到9的每一個數字。

教師雜誌倡引入課堂

英國現在到處有人談論「數獨」；首本關於「數獨」的月刊已於周前推出；多本關於「數獨」的書在市場上出售；電視台亦正在籌備相關的電視節目；政府資助的《教師》雜誌更建議把「數獨」引入課堂，作爲學生鍛煉腦力的遊戲。就連英國廣播公司第4頻道也製作節目讓聽衆參與玩「數獨」遊戲。《觀察家報》形容，「數獨」就如「21世紀的扭計骰」。扭計骰在1981年推出後，曾風靡全球，至今銷量達到1億。

英報紛刊「數獨」搶銷量

「數獨」還成爲英國報章增加銷量的法寶，多份全國性報章紛紛針對它大做文章。《衛報》周五在頭版宣稱：「G2（《衛報》副刊）——唯一一份每頁都印有『數獨』的報章」。《泰晤士報》則大字標題報道：「『手機數獨』：每個人都在談論的遊戲，現在出現在你的手機上了」。《獨立報》亦在底版刊載一個簡單的「數獨」遊戲，並在一個內頁再刊載3個不同難度的版本。

在歐洲其他地方，「數獨」也愈益受到歡迎，美國高中亦流行起「數獨」卡片遊戲。上月，美國《紐約郵報》開始定期刊載「數獨」遊戲。《觀察家報》稱，這意味「數獨」完成了「環球之旅」。 觀察家報/獨立報/星期日電訊報

「數字拼圖」遊戲玩法（入門）

		1				8		
	7		3	1			9	
3				4	5			7
	9		7			5		
	4	2		5		1	3	
		3			9		4	
2			5	7				4
	3			9	1		6	
		4				3		

（答案見版底）

遊戲規則：
在每個空格中填寫數字1至9，令大正方形中每一行、每一列，以及每個由粗線隔開的小九宮格中，均包括數字1至9

讀者可到以下網站試玩
www.puzzle.jp/letsplay/play_sudoku-e.html
www.sudoku.com

智力遊戲迷 一見傾情努力研究

「數獨」遊戲在20餘年前便在日本大受歡迎，但真正令它「全球化」的，卻是前香港高等法院法官高樂德（右圖）。高樂德去年把遊戲推銷給英國《泰晤士報》，成功掀起熱潮。

花6年時間 設計遊戲電腦程式

高樂德本身是一名智力遊戲迷。1997年3月，他在臨退休前到東京旅遊，在銀座一書店隨手拿起一本智力遊戲書，隨即被「數獨」遊戲吸引。「我一看到大正方形裏的空格，便很想填滿它們。」高樂德隨後花了6年時間，設計出一個能產生出不同「數獨」版本的電腦程式。他去年接受《泰晤士報》訪問時說：「我輸入了2.5萬個獨立的數字拼圖，確保全部能被它（電腦程式）解答。對我來說，這是個全新起點，在正常情況下，你不會想像法官精於寫電腦程式，對不對？」

高樂德首先在一份美國地方報紙上刊登他設計的一個「數獨」遊戲，獲得熱烈回響。去年底，「數獨」熱開始在英國出現。他憶述：「我（當時）正途經倫敦前往香港。我像一名傳銷推銷員一樣，在沒有通知下走進《泰晤士報》，邁出了第一步。他們在接下來那個月刊出遊戲，大受歡迎。」高樂德現在以香港爲基地開設網站www.sudoku.com，出售「數獨」電腦遊戲，由法官搖身一變成創業家。 泰晤士報/觀察家報

日人改良 遊戲重見天日

「數獨」遊戲其實已有逾200年歷史。1783年，瑞士數學家奧伊勒（Leonhard Euler）發明了它的前身「拉丁方形」（Latin Squares）。「拉丁方形」與「數獨」的唯一不同之處，是它並沒有再細分成9個九宮格。

1970年代末期，美國益智雜誌Dell Puzzle Magazines開始刊載「拉丁方形」，稱爲「數字拼圖」。日本益智雜誌的職員金元信彥相信這個遊戲有助於擴大雜誌的讀者群，決定將它引入日本。他說：「『數獨』的奇妙之處在於它容易令人着迷。雖然它有難度，但你就算會讀小說讀到發悶，也不會覺得『數獨』無聊。」經日本人改革提高難度的「數獨」，很快在日本掀起熱潮。 星期日電訊報

		1				8		
	7		3	1			9	
3				4	5			7
	9		7			5		
	4	2		5		1	3	
		3			9		4	
2			5	7				4
	3			9	1		6	
		4				3		

The Su-do-ku Craze

Financial Times
2005.05. 28/29

Count me in on the Sudoku number puzzle craze

I first came across Sudoku a couple of Saturday mornings ago, when I wandered down to the corner shop to buy a weekend newspaper, *writes Stephen Pincock.*

The folks who run my excellent "local mixed business" stock most of Britain's plethora of daily national papers, which they arrange in convenient piles on a low wooden platform near the front door. Bending down to grab the FT (naturally) and some other rag, I noticed the word "Sudoku" on the other paper's front page.

It sat, unexplained, in a prominent spot at the top right-hand corner of the page. Glancing along the pile, I saw the word crop up above the fold on a surprising number of the other papers. One touted its handcrafted "classic Sudoku", another had three puzzles of varying complexity, while yet another claimed to have been the first to introduce Sudoku to the UK. What on earth, I thought, are these things?

It didn't take long to find that they are number puzzles laid out on a nine by nine grid of squares, some of which contain numbers between one and nine. The idea is to fill in the blank squares in such a way that each row, each column and every three-by-three square within the larger grid contains every number between one and nine, once only.

A quick search on the internet revealed that the Sudoku fad came to Britain via Japan, where the puzzles have been popular since the 1980s. The "su" in the name stands for number, while "doku" connotes singular. In Japan, the name Sudoku is the copyright of Nikoli publishing, although the puzzle seems to have been published first in the US, where it is called the Number Place puzzle.

In fact, at a stretch, you can trace the origins of Sudoku back to something invented in the 18th century by one of history's greatest mathematicians, Leonhard Euler. Euler was a Swiss-born child prodigy who went on to become one of the most prolific mathematicians of all time. In 1783, he introduced the idea of "Latin squares", which he called a "nouveau espèce de carrés magiques" – a new kind of magic squares. Latin squares are grids of any size in which each number appears once in each column and every row, just as in Sudoku puzzles.

Regardless of their heritage, I've learned that Sudoku can be compulsive. It didn't take too many hours after that trip to the newsagents before my wife was wondering whether she was becoming a "Sudoku widow".

The rules are simple, although they can become fiendishly difficult as the number of given numbers drops. But the odd thing is that newspapers make a point of saying that the puzzles require no mathematics.

The assumption seems to be that maths is nothing more than arithmetic – addition, subtraction, multiplication and division. This reflects a complete misunderstanding of what the subject is all about, says Charles Leedham-Green, professor of pure mathematics at Queen Mary, University of London. "One thing that mildly irritates me is this idea that because you don't have to add the numbers up in Sudoku then it's not mathematics."

In fact, the process involved in figuring out how to solve number placement puzzles is very mathematical, according to Leedham-Green. "Thinking of a way to tackle a Sudoku is a mathematical problem. Obviously, no one thinks it is high mathematics, but the process of coming up with the tricks to solve these puzzles is mathematical."

Marcus du Sautoy, professor of mathematics at the University of Oxford, agrees. In fact, he sees parallels between the thrill of beating a Sudoku puzzle and the excitement of mathematical breakthroughs. "I think the buzz people get from finishing them is the same kind of buzz I get from mathematics," he says.

The thing is, mathematicians don't spend their days working through page after page of mind-numbing arithmetic. Rather, they bend their minds to finding ways to solve problems that are often deeply conceptual. They approach those challenges using the kind of strategic thinking you need to employ in tackling Sudoku puzzles.

You might not know it, but the decisions [illegible] make in finishing a [illegible] fall into the category of heuristics, the discipline of solving problems using intelligent choices when applying a formula is impossible. There are other maths terms that might also apply – a mathematician might think of Sudoku as an exercise in "graph colouring", and consider a strategy for tackling it as a "backtrack search".

The puzzles themselves have even triggered their own mathematical conundrum – the number of possible valid arrangements is currently unknown, despite the fact that mathematicians have figured out the number of possible variants of a Latin square that's nine by nine.

I'm not trying to baffle you with terminology here, but to point out that Sudoku might teach us something unexpected about maths and mathematicians. Put aside the prejudices developed during double-maths on a sunny Friday afternoon and consider this idea – maths might actually be fun.

stephen.pincock@journalist.co.uk

Count me in on the Sudoku number craze

Stephen Pincock

Financial Times

May 28, 2005.

Count me in. The real puzzle behind Sudoku is the idea that maths doesn't come into play. Well, the number crunchers will have the last laugh.

Stephen Pincock

***Financial Times*, May 28, 2005.**

"One thing that mildly irritates me is this idea that because you don't have to add the numbers up in Sudoku then it's not mathematics."

Charles Leedham-Green,
Professor of Pure Mathematics
at Queen Mary College, University of London.

「每位專業數學家都清楚知道娛樂成份與認真態度並不相悖。主要功夫是保證讀者既歡享娛樂成份卻不會忽略數學上的重點。」

Preface in : J.B. Baylis, R. Haggarty,
Alice in Numberland: A Students' Guide To the Enjoyment of Higher Mathematics (1988)

嬰兒生無石師而能言，
與能言者處也。

莊子・外物（公元前四世紀）

[When a child is born, it needs no great teacher ; nevertheless it learns to talk as it lives with those who talk.]

Zhuangzi (Chuang Tzu)

Book 26 : Affected from Outside

(4th century B.C.E.)

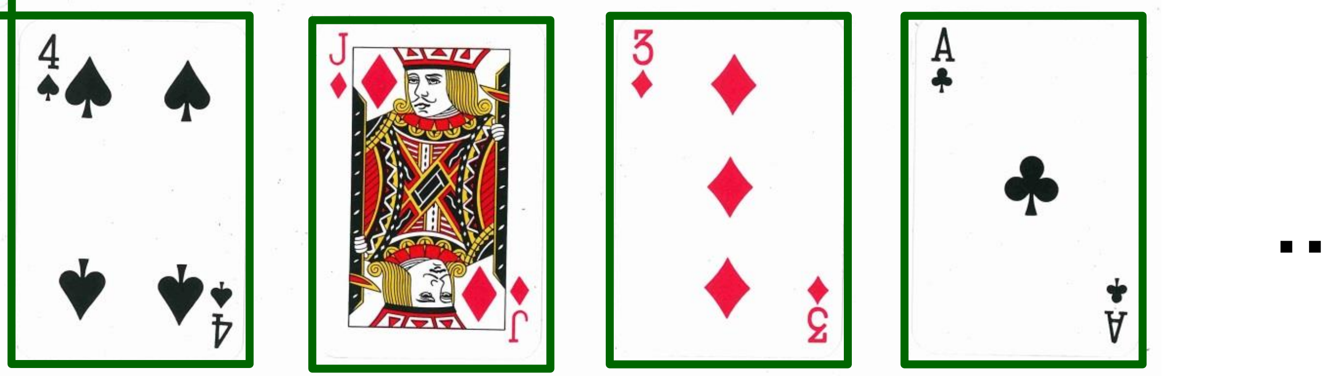
「我們不是因為年老而停止遊戲，而是因為停止遊戲才會變老。」

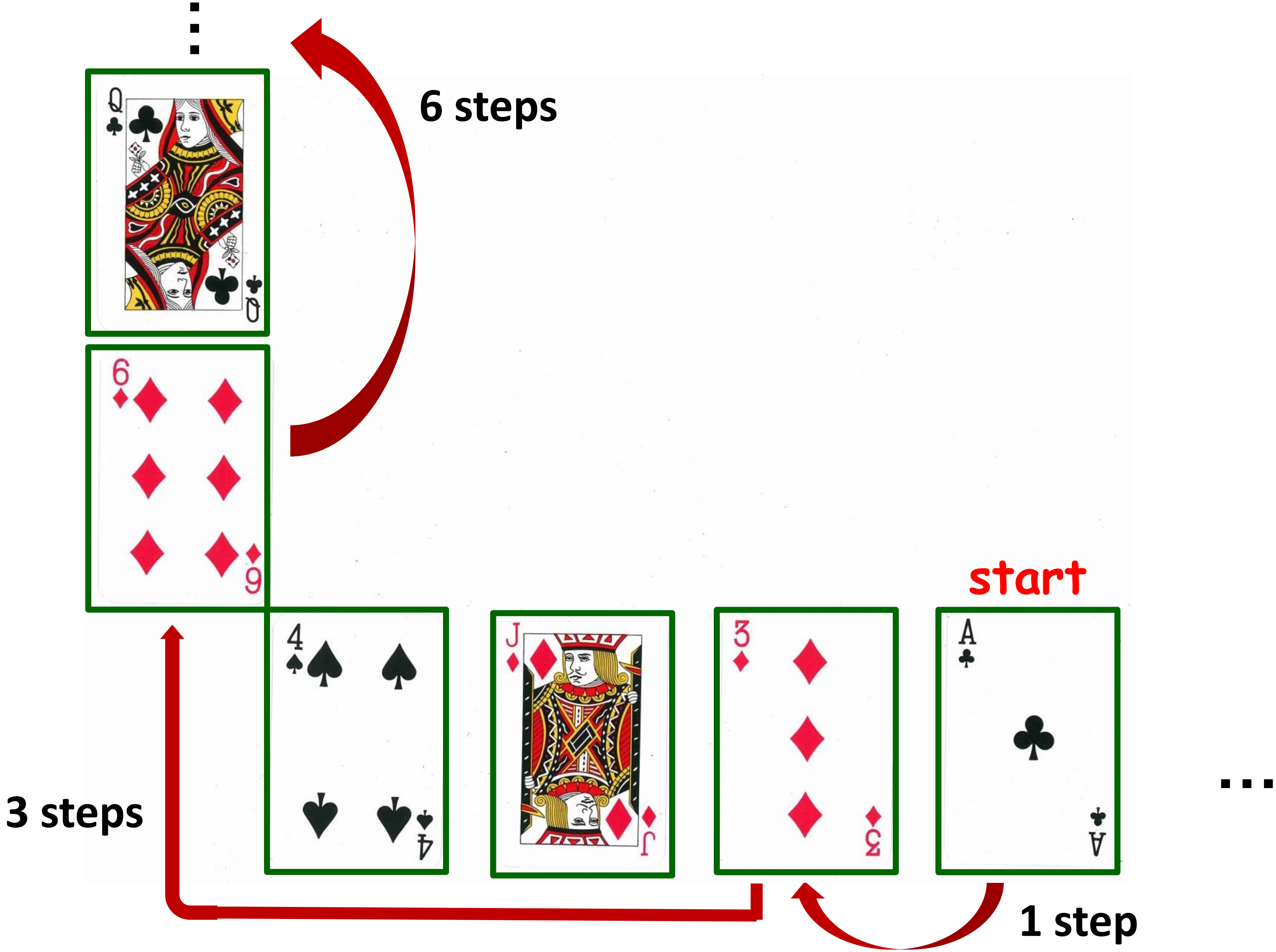
（這句話的來源眾說紛紜，有些人認為是富蘭克林 [Benjamin Franklin 1706-1790] 說的，有些人認為是蕭伯納 [George Bernard Shaw 1856-1950]說的，還有別的說法。）

「[Karl] Groos 說得好，他說孩子年輕，是因為他們玩耍，並非因為他們年輕才玩耍；其實，他可以加上一句：人變老，是因為他們停止玩耍，也非他們年老便停止玩耍。因為，說到底，玩耍就是成長，而且在智力最高層次上，它是源於摯愛真理而作的永恆探索。」

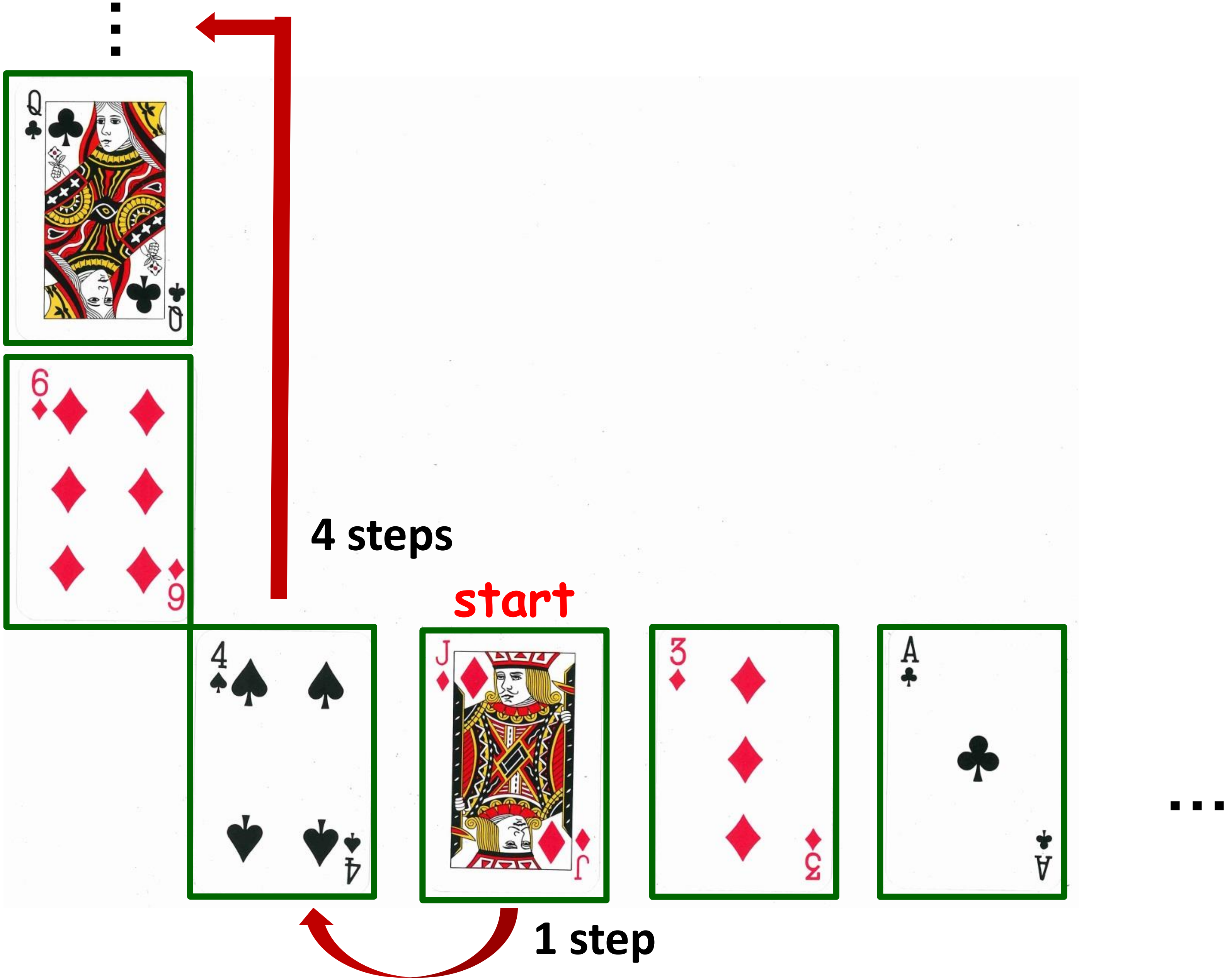
（這句話來在來自美國心理學家霍爾 [Granville Stanley Hall 1844-1924]的名著《青春期：它的心理學及其與生理學、人類學、社會學、性、犯罪、宗教和教育的關係》[*Adolescence: Its Psychology and its Relations to Physiology, Anthropology, Sociology, Sex, Crime, Religion and Education*, D. Appleton & Company, 1904] ）

A magic card trick based on the "Kruskal's Count" credited to the mathematician-physicist Martin David Kruskal (1925-2006)





Start with club Ace.



Start with Diamond Jack.

	2	4	1	3	2	2	1	4	
1									3
3									3
1									4
4									1
2									2
1									3
2									1
4									3

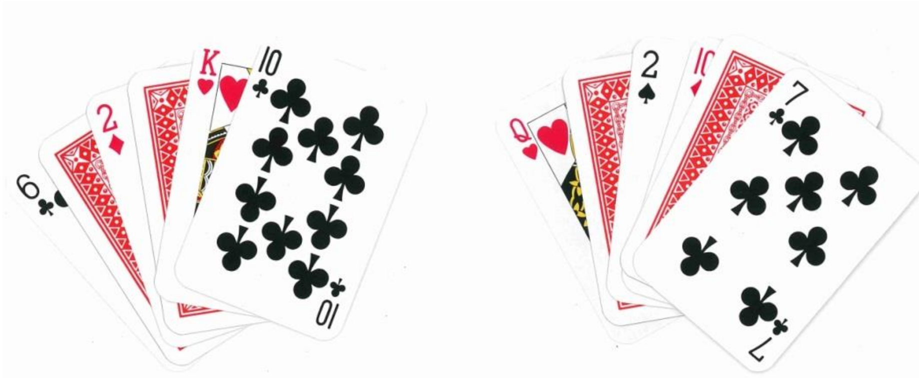
HOME

1	3	4	2	3	4	2	4

Start with **any** card in HOME and count your steps by the next number you land on.

Stop when you **first** get back to HOME.

Shuffle a pack of cards, with half of them face-up and half of them face-down.
Divide the pack in half with the same number of cards face-up and face-down in each half.



4 face-up	**4 face-up**
2 face-down	**2 face-down**

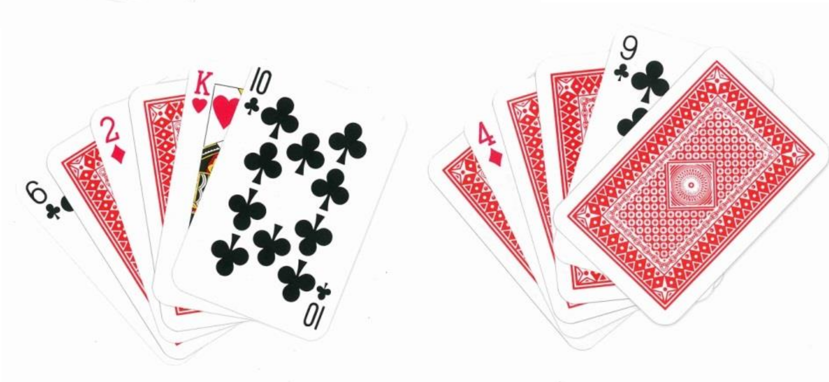
A magic card trick invented by Bob Hummer

[See: Chapter 1, Martin Gardner, *Mathematics, Magic and Mystery* (1956)]

In general …

N **face-up**

N **face-down**

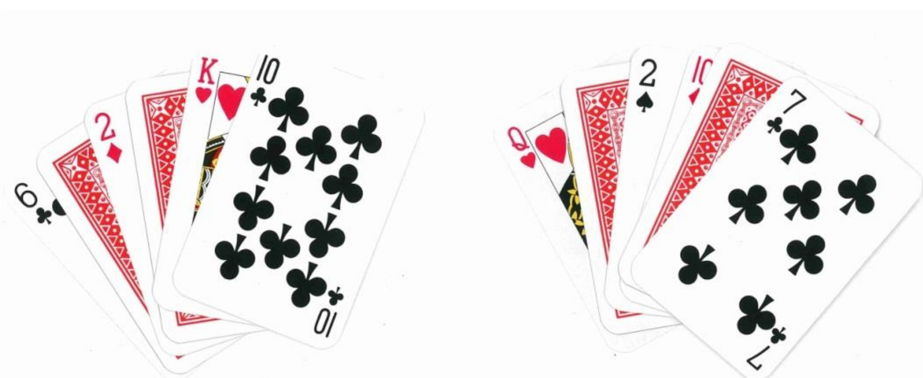


a **face-up**

N - a **face-down**

N - a **face-up**

a **face-down**

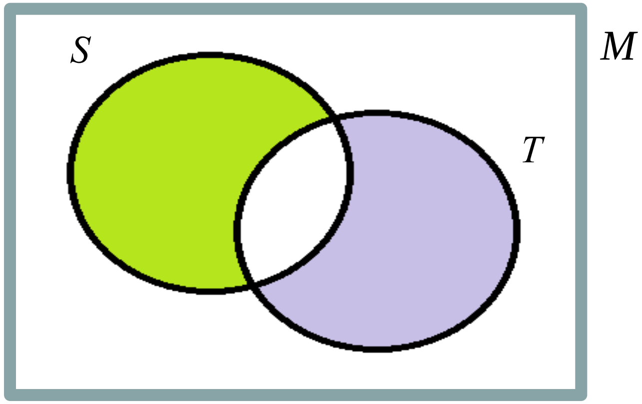


a **face-up**

N - a **face down**

a **face-up**

N - a **face-down**



If $m(S) = m(T)$,
then $m(S \backslash T) = m(T \backslash S)$.

M = the set of all 52 cards;

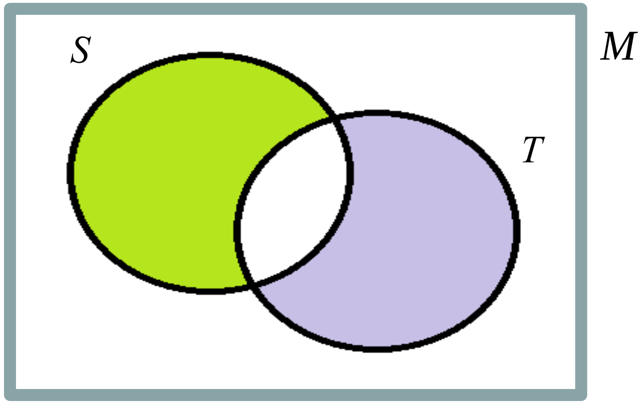
S = the set of 26 face-up cards;

T = the set of 26 cards in one pile (A).

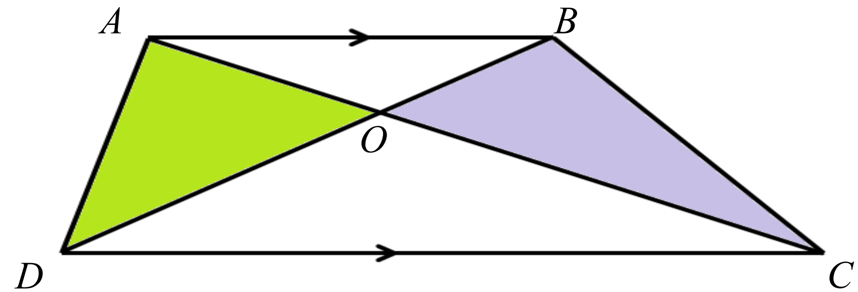
Then $(S \backslash T)$ = the set of face-up cards in the other pile $(M - A)$;

$(T \backslash S)$ = the set of face-down cards in the pile A.

By reversing one of the two piles, the number of face-up cards in each will be the same.



If $m(S) = m(T)$,
then $m(S\setminus T) = m(T\setminus S)$.

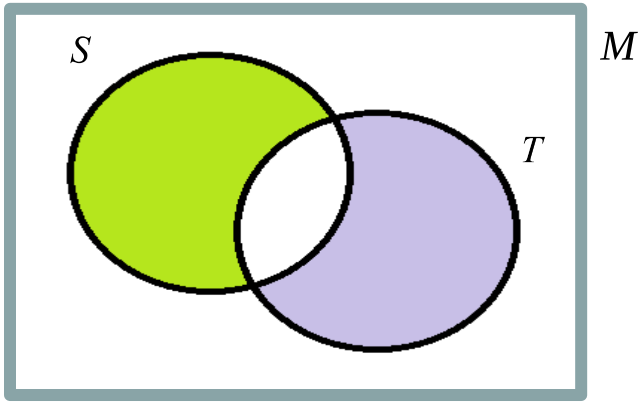


$ABCD$ is a trapezium with AB parallel to DC.
AC and BD intersect at O.
Then $\triangle AOD$ and $\triangle BOC$ have the same area.

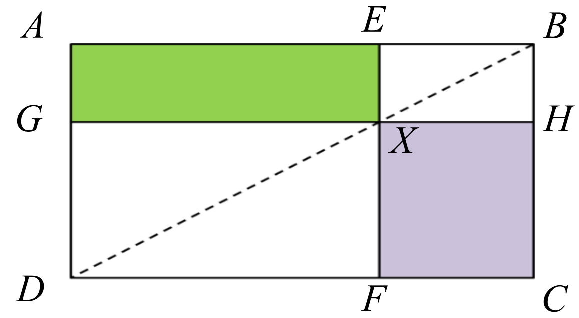
Let $S = \triangle ACD$;

$T = \triangle BCD$.

Since $\triangle ACD$ and $\triangle BCD$ have the same area, it follows that $\triangle AOD$ and $\triangle BOC$ have the same area.

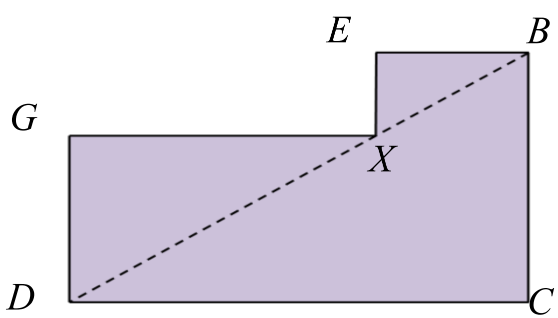


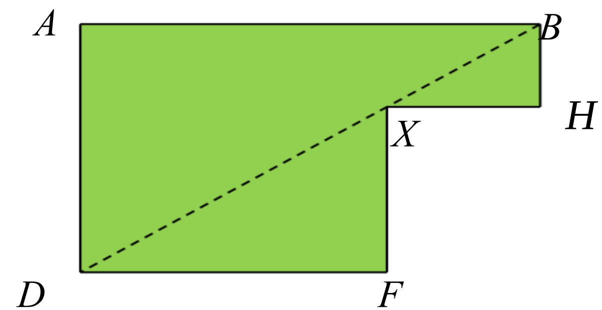
If $m(S) = m(T)$,
then $m(S \setminus T) = m(T \setminus S)$.



$ABCD$ is a rectangle, and X is a point on the diagonal BD.
EXF is a parallel to AD; GXH is parallel to AB.

Then the rectangles $AEXG$ and $XHCF$ have equal area.





Let $S = CDGXEB$

Let $T = ABHXFD$

Since $CDGXEB$ and $ABHXFD$ have the same area, it follows that $AEXG$ and $XHCF$ have the same area.

勾(股)中容橫。股(勾)中容直。

二積皆同。古人以題易名。

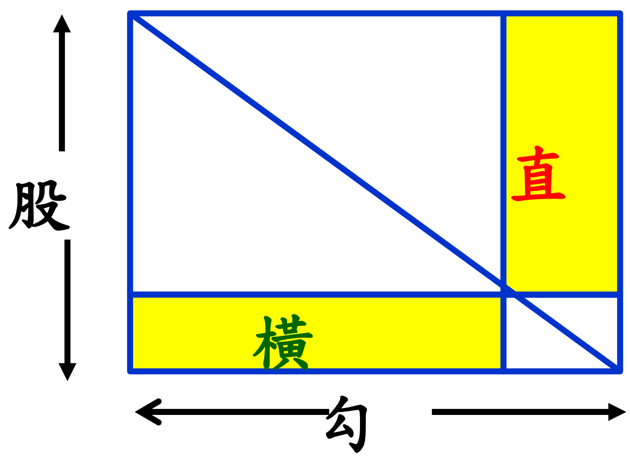
若非釋名。則無以知其源。

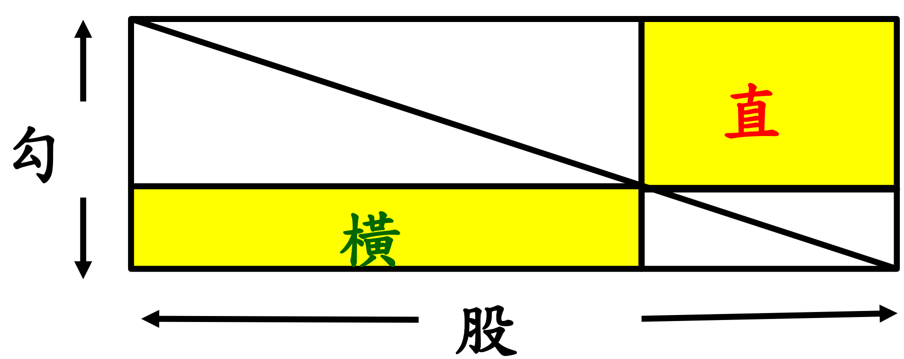
(The horizontal rectangle formed by part of the base and the vertical rectangle formed by part of the perpendicular are equal in area. Men of the past changed the names of their methods from problem to problem …)

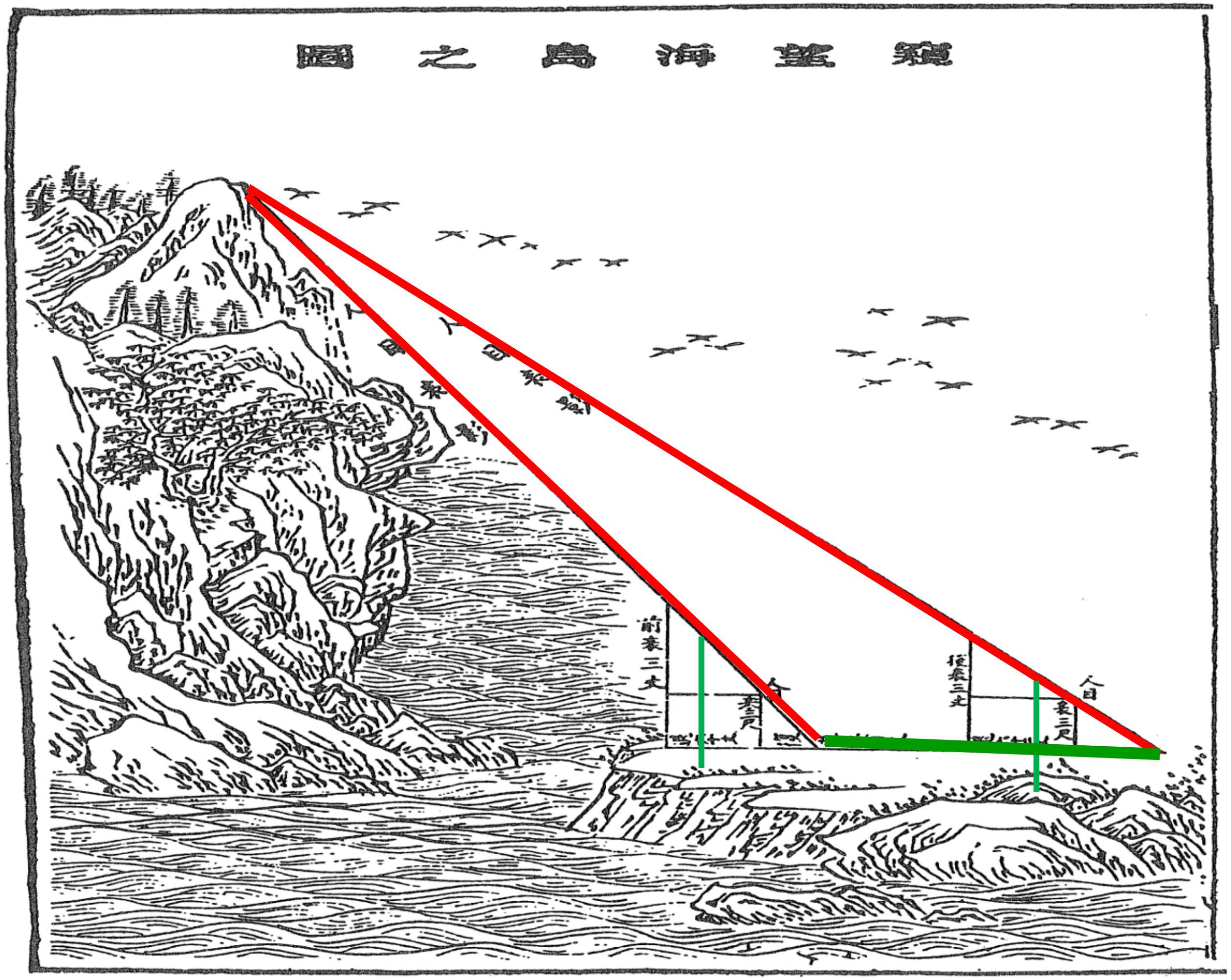
Compare with Proposition 43 of Book I of Euclid's *Elements.*

楊輝，《續古摘奇算法(卷下)》

YANG Hui, *Continuation of Ancient Mathematical Methods for Elucidating the Strange* [*Properties of Numbers*] (Chapter II) (1275)

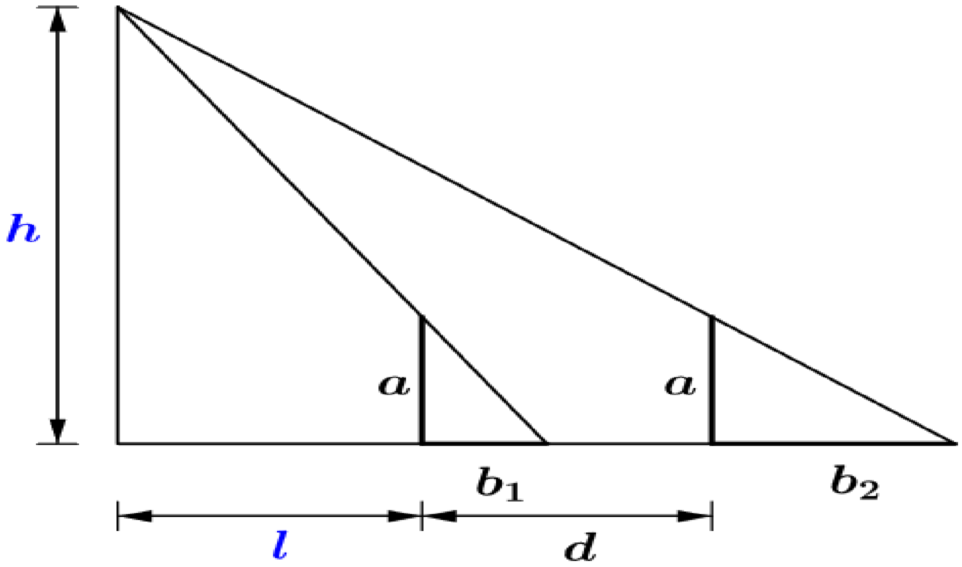






LIU Hui's Method of Double-Difference
in *Haidao Suanjing* [海島算經 Sea Island Mathematical Manual]
(3rd century) as illustrated in *Gujin Tushu Jicheng*
[古今圖書集成 Complete Collection of Pictures and Writings
of Ancient and Modern Times] (1726)

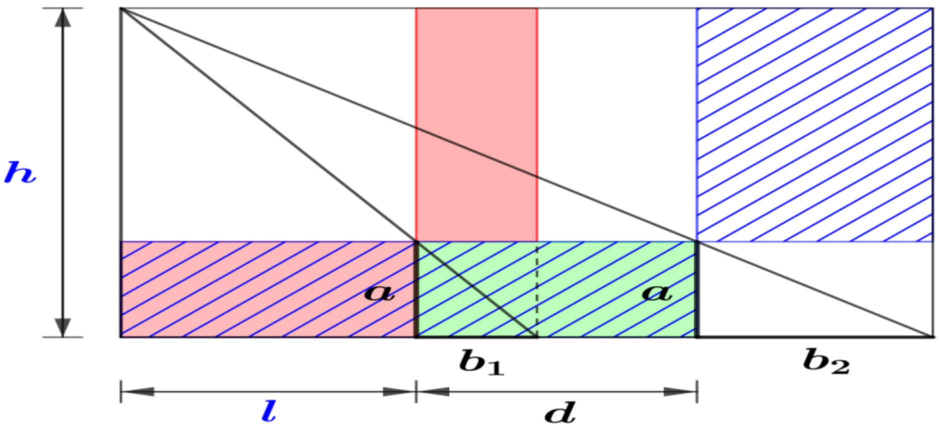
Given a, d, b_1 and b_2, how can we express h and l in terms of a, d, b_1 and b_2 ?

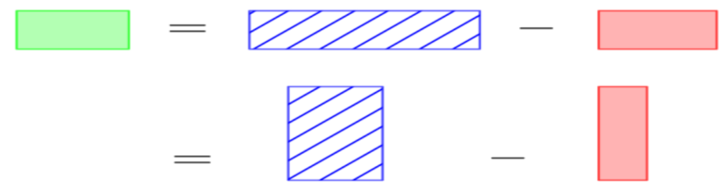


http://ggbtu.be/m2812113

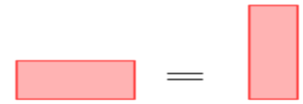
Explanation by YANG Hui on the Method of Double-Difference of LIU Hui (1275)

Given a, d, b_1 and b_2, how can we express h and l in terms of a, d, b_1 and b_2 ?





$$\therefore \quad ad = (h-a)b_2 - (h-a)b_1$$
$$= (h-a)(b_2 - b_1)$$
$$h = \frac{ad}{b_2 - b_1} + a$$

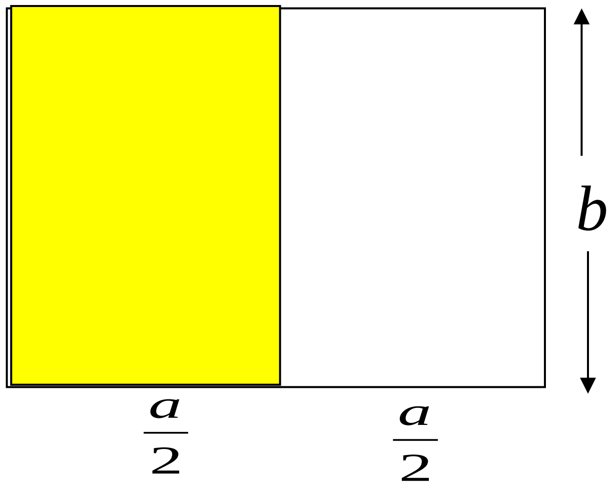


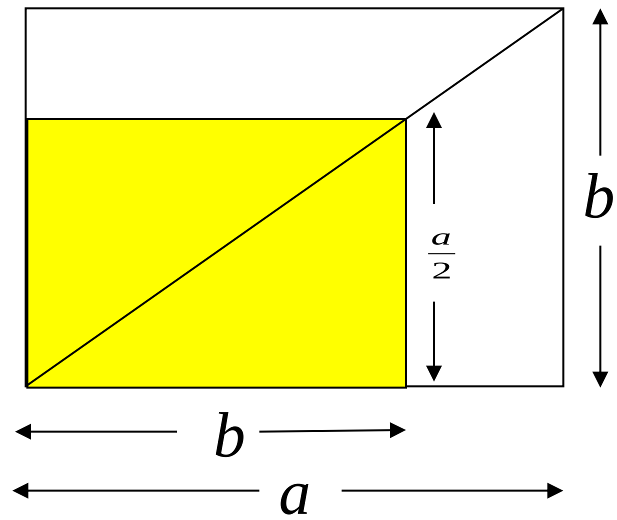
$$\therefore \quad la = b_1(h-a) = \frac{b_1 ad}{b_2 - b_1}$$
$$l = \frac{b_1 d}{b_2 - b_1}$$

http://ggbtu.be/m2812113

Explanation by YANG Hui on the Method of Double-Difference of LIU Hui (1275)

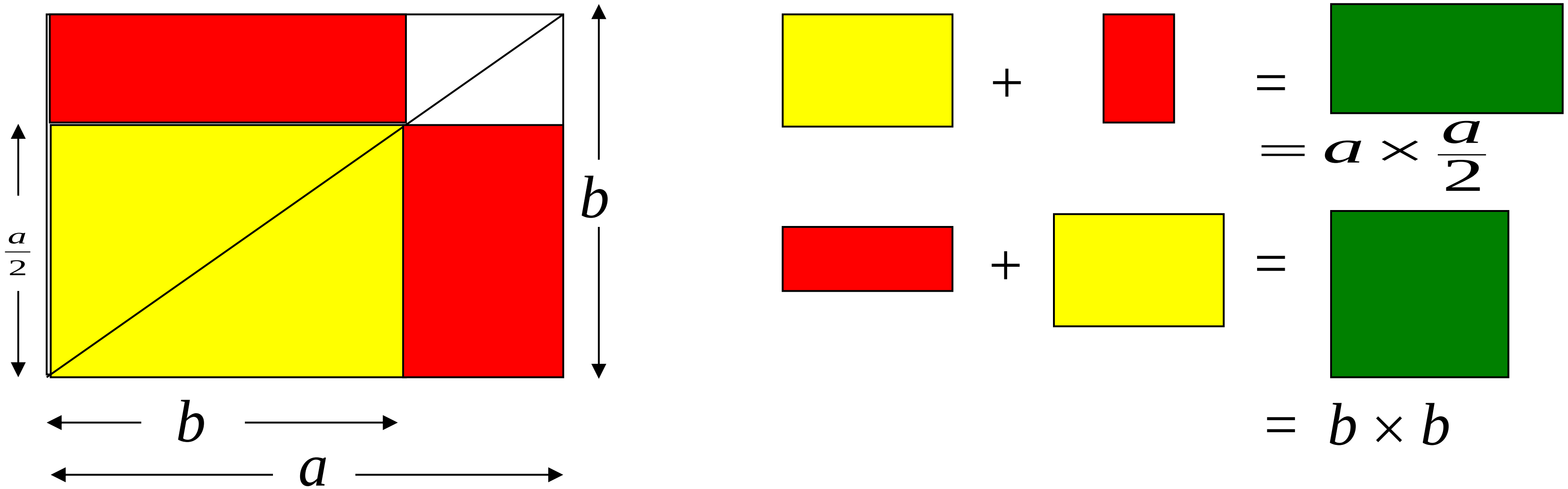
Size of A3 and A4 paper





$$a = ?\ ,\ b = ?\ .$$

Factor of enlargement and reduction in a photocopier



$$a \times \frac{a}{2} = b \times b$$

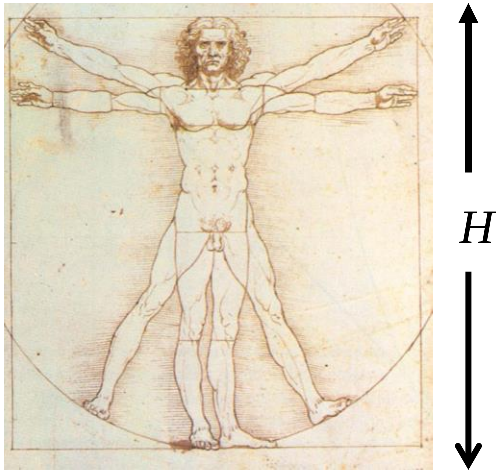
$$a \times a = 2 \times b \times b$$

$$a = \sqrt{2} \times b$$

Magnifying factor = (√2) × 100 % ≈ 141 %

Shrinking factor = (1/√2) × 100 % ≈ 71 %

A man weighing 50*Kg* can normally lift up 30*Kg* . How much can a man weighing 100*Kg* normally lift up?



Weight (W) is proportional to the cube of height.

Weight capable of lifting up (F) is proportional to the cross-sectional area of the muscle, hence proportional to the square of height (H).

$$\frac{W_1}{W_2}=\frac{H_1^{\,3}}{H_2^{\,3}},\quad \frac{F_1}{F_2}=\frac{H_1^{\,2}}{H_2^{\,2}},$$

Therefore $\boxed{\dfrac{W_1^{\,2}}{W_2^{\,2}}=\dfrac{F_1^{\,3}}{F_2^{\,3}}}$.

If $W_1=2W_2$, then $\dfrac{F_1^{\,3}}{F_2^{\,3}}=4$, $\dfrac{F_1}{F_2}=\sqrt[3]{4}=1.5874...$

Since F_2 is $30Kg$, F_1 is computed to be $47.62...Kg$.

An ant normally measures 0.005m. It can carry a burden that is 5 times its own weight. If a giant ant were as big as a man (say of height 1.75m), how much times of its own weight would it be able to carry?

$$H_2 \qquad\qquad H_1 = \frac{1.75}{0.005} H_2 = 350 H_2$$

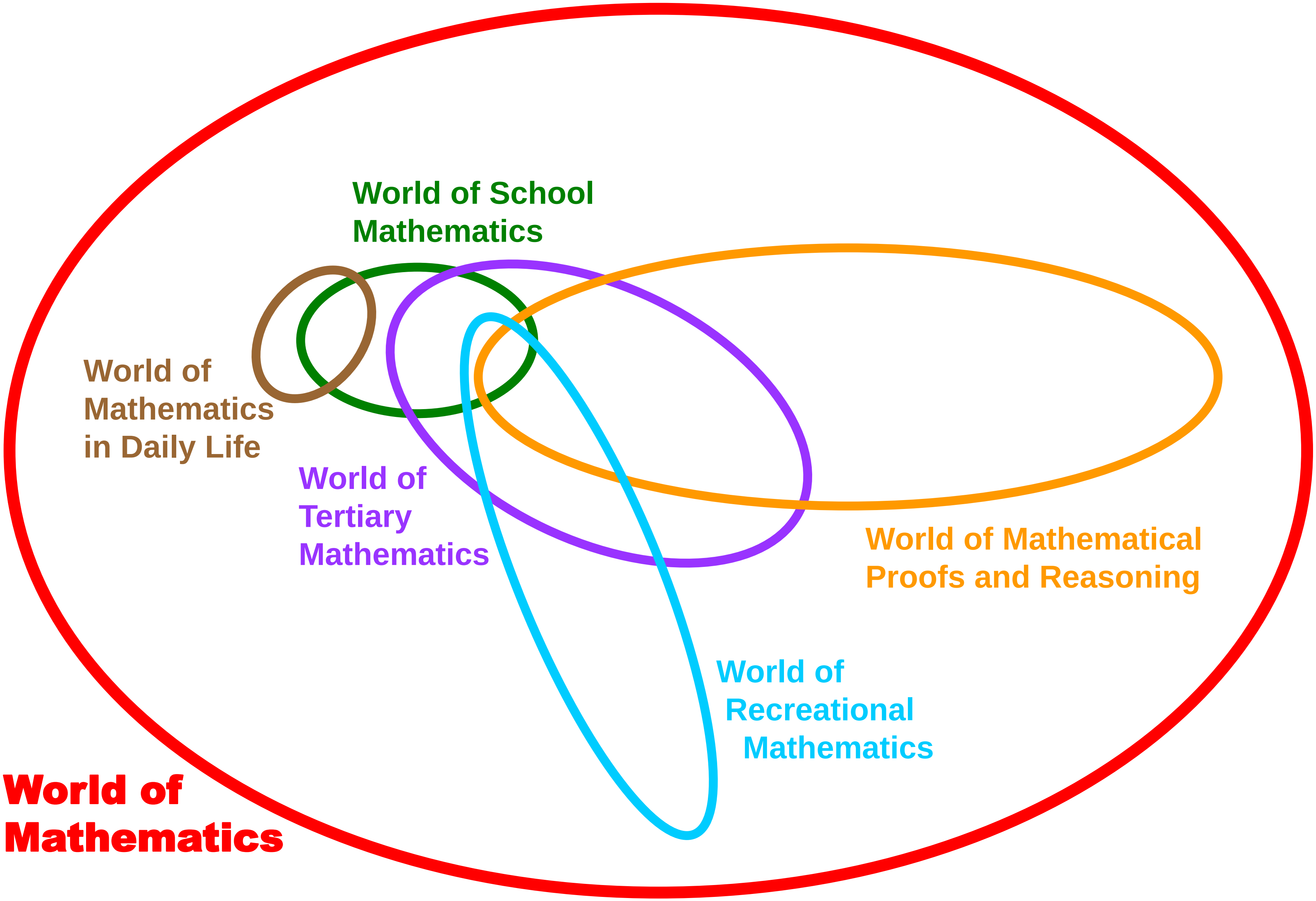
$W_1 = 350 \times 350 \times 350 \times W_2$, $F_1 = 350 \times 350 \times F_2$,

It is known that $\frac{F_2}{W_2} = 5$,

therefore $$\frac{F_1}{W_1} = \frac{350 \times 350 \times F_2}{350 \times 350 \times 350 \times W_2}$$

$$= \frac{1}{350} \times \frac{F_2}{W_2} = \frac{1}{70},$$

The giant ant can only carry 1/70 of its own weight. It can hardly stand on its own feet!



World(s) of Mathematics

Is mathematics

— a useful science?

— a vibrant science?

— an amusing science?

— a rigorous science?

— a heuristic science?

— an experimental science?

— a humane science?

Or even, is mathematics a subject in science or in arts ?

THE BLIND MEN AND THE ELEPHANT
John Godfrey Saxe (1872), based on an ancient Indian fable

The first felt the side of the elephant and said: It's a wall.
The second got hold of a tusk and said: It's spear.
The third felt the trunk and said: It's a snake.
The fourth put his arms around a leg and said: It's a tree.
The fifth touched the ears and said: It's a fan.
The sixth grasped the tail and said: It's a rope.

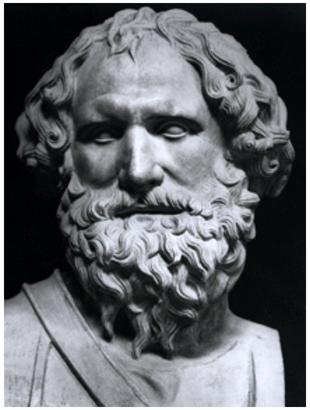
**"Though each was partly in the right,
And all were in the wrong!"**

John Godfrey Saxe (1816-1887)

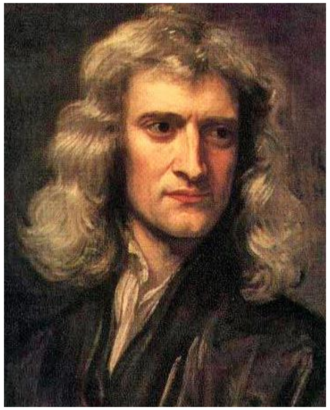
「依我看，STEM 不是一個學科，也更加不僅是一堆現代科技產品。STEM 是一種綜合意識，滲透在不同的學科，以學習和運用數學知識及科學知識，再以工程手段配合現代科技，改善生活。STEM 蘊含一種探索精神及思想方式，揉合了數學思維及科學精神，通過實驗、觀察、理論整理以尋求知識，進而創新。」

蕭文強,推薦序,
盧安迪,《STEM教育與美國》(2018), 頁10.

STEM
ANEG



Archimedes
(287–212 B.C.E.)



Isaac Newton
(1642-1727)

Leonhard Euler
(1707-1783)

Carl Friedrich Gauss
(1777-1855)

「度數旁通十事」：

「其一（天氣），其二（測量），其三（樂律），
其四（軍事），其五（會計），其六（建築），
其七（機械），其八（輿圖），其九（醫學），
其十（時計）。

「右十條於民事似為關切。
臣聞之周髀算經云：禹之所以
治天下者，句股之所繇生也。
蓋凡物有形有質，莫不資於
度數故耳。」

徐光啟，條議曆法修正歲差疏, 1629

徐光啟
XU Guang-qi
(1562-1633)

STEM教育系列(一) 中、小數學課程如何迎向科學、科技、工程和數學教育？

- **STEM 遇上 GeoGebra 之研討會暨工作坊 (香港 GeoGebra 學院合辦)**
  日期： 2016 年 6 月 11 日
  講者:柯志明
  回應 / 討論：李文生

- **圖像顯示計算機於數學教學的應用：STEM 教育的課堂例子兼談新加坡數學教育的改革**
  日期： 2016 年 6 月 24 日
  講者: Ng Wee Leng, 黎凱源, 鄧惠子, 廖國威, 周偉麟, 鄭德森
  回應 / 討論： 鄧國俊

- **STEM 教育在中、小學數學課程的推行現況、計劃和問題**
  日期： 2016 年 7 月 2 日
  講者:葉葆誠, 鄧佩玉, 潘維凱, 鄧國俊
  回應 / 討論：李文生

STEM教育系列(二) 從數學教育看STEM教育

- **從 GeoGebra 到 gMath：科技可為我們帶來甚麼？**
  日期：2017 年 11 月 18 日
  講者:戚文鋒
  回應 / 討論：郭嘉欣, 潘維凱

- **STEM 所帶來的機遇與挑戰：化擔心為力量**
  日期：2017 年 12 月 9 日
  講者:鄧國俊, 黃家樂

- **以數學科為主體的STEM：由實驗課程「數形探極」談到小學數學科的定位**
  日期：2018 年 1 月 6 日
  講者:李文生, 譚志良, 馮振業

- **STEM在學校的推行：數學老師的角色**
  日期：2018 年 1 月 27 日
  講者:葉碧君, 潘維凱, 梁健儀

STEM教育系列(三) 數學中的STEM教育

- 「數學作為科學」（Maths As Science）
  日期：2018 年 11 月 24 日（星期六）
  講者: 羅浩源
  回應/討論 :曾建勳, 麥建偉

- 「數學作為工程」（Maths As Engineering）
  日期： 2018 年 12 月 8 日（星期六）
  講者:羅浩源
  回應 / 討論：徐崑玉, 潘維凱

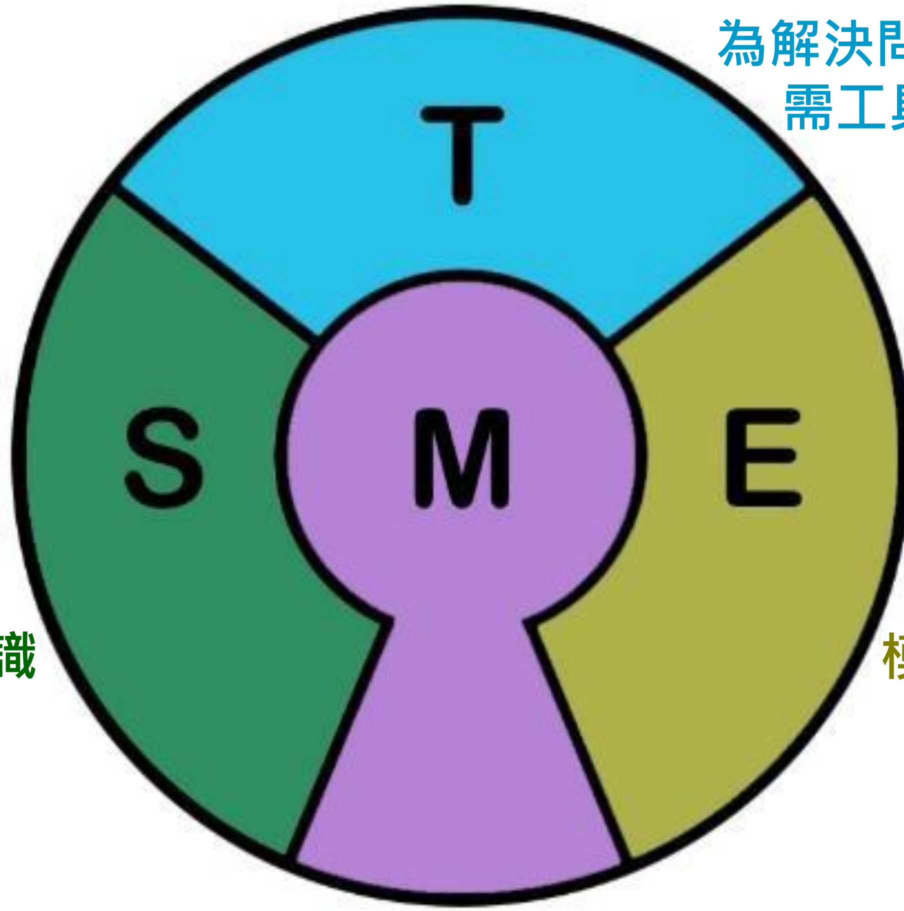
- 「數學作為科技」（Maths As Technology）
  日期： 2019 年 1 月 19 日（星期六）
  講者:羅浩源
  回應 / 討論：鄧佩玉, 楊鳳興

STEM 教育系列（三） 數學課程中的 STEM 教育

新修訂的數學課程將會由第一學習階段（即小一至小三），於 2019/20 學年起在小一逐年推行，其中內容配合著正在香港推動的 STEM 教育。STEM 教育本身包含了發展學生綜合和應用科學、科技、工程和數學的知識和技能上的培育，但究竟要加入甚麼元素和如何加入才能把 STEM 教育在數學課堂中實踐？從發展數學教育的角度來看，又為何需要重新評估和創造數學在 STEM 的角色？我們希望通過這系列的講座，與參與者一起探討數學課程中發展 STEM 教育的挑戰和機遇。

The Role of Maths in STEM

為解決問題創造所
需工具的意識



探索自然
現象的意識

設計解難
模型的意識

以數學語言理解生活經驗的意識

羅浩源，STEM 教育：以數學作起點來推動STEM 教育的挑戰，
School Mathematics Newsletter, Issue 21 (2017), pp. 6-11.

SMET

Science
Mathematics
Engineering
Technology

It was said that in 2001 Judith Ramaley, then Assistant Director for Education and Human Resources at NSF, thought that SMET does not sound as good as STEM, so she changed the acronym to STEM!

STEM

Science
Technology
Engineering
Mathematics

STEM	STEAM
Science	Science
Technology	Technology
Engineering	Engineering
Mathematics	Arts
	Mathematics

STEM	STREAM
Science	Science
Technology	Technology
Engineering	Reading
Mathematics	Engineering
	Arts
	Mathematics

STEM	iSTREAM
Science	information science
Technology	Science
Engineering	Technology
Mathematics	Reading
	Engineering
	Arts
	Mathematics

STEM	STREAiM
Science	Science
Technology	Technology
Engineering	Reading
Mathematics	Engineering
	Artificial intelligence
	Mathematics

STEM	THAMES
Science	**T**echnology
Technology	**H**umanities
Engineering	**A**rts/**A**rtificial intelligence
Mathematics	**M**athematics
	Engineering
	Science

加上一份人文關懷，想想人類的悠長歷史及其興衰成敗，由此學懂謙卑包容。

科學並非萬能，亦非統領一切，我們必須學懂如何與人相處，與自然共存。

THAMES

Technology

Humanities

Arts/**A**rtificial intelligence

Mathematics

Engineering

Science

教學就是說故事，要說一個好故事，一個能引起好奇和激發想像的好故事，一個關於人類在悠長歲月探索理解周遭世界的故事。[…]

數學是文化的一部份，它並不只是工具而已，那怕已經證明了它是非常有用的工具；因此，數學的發展歷史，以及由古至今數學與其他人類的奮鬥活動的關係，都應該是這學科的一部份。[…]

就是因為數學能有機地融入其他知識和文化活動，數學科才成為更值得學習的科目。在這樣更廣泛的層面來說，數學史更加肩負全人教育的一個重要任務。

蕭文強, 數學可以怎樣教得更好？
《數學傳播》[*Mathmedia*] , 40 (1) (2016), 81-86.
[Chinese translation of a presentation in a Plenary Panel at the ICM-2020, Seoul, South Korea, August, 2014]

Thanks to the HKAME and the OUHK for their invitation to give this plenary lecture!

Any comment, suggestion or question are welcome. Please contact me at mathsiu@hku.hk .

I like to thank Ms. Mimi Lui of the HKU Department of Mathematics for her help with the powerpoint slides, and Mr. Or Chi Ming for his help with the *GeoGebra* applets.